

LUNCHMATHS

1 pm, Tuesday May 11th, 2004
Lecture Theatre S14

What is $E(X)$ really?

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One of the most painful lectures which first year students have to endure is the lecture on Riemann Sums. We could spare them this pain by saying: *If f is continuous on $[a, b]$, we define the integral of f over $[a, b]$ by the formula:*

$$\int_a^b f(x) dx = F(b) - F(a), \quad (1)$$

where $F' = f$. By the way, when $f \geq 0$, this represents the area under the curve. Yes, it really does! You can trust us. We are mathematicians.

Now let X be a continuous random variable. We define the expected (or mean or average) value of X by the formula

$$E(X) = \int_a^b f_s(x) dx = F(b) - F(a), \quad (2)$$

Where f_s is the density function of X . Yes, this really is a deep and meaningful concept! You can trust us. We are statisticians.

The speaker will revive a most ancient method called the “method of exhaustion”. Riemann sums provide an instance of this method. Archimedes used his own version of the method to obtain the area of certain regions with curved boundaries (circles and parabolic segments). If such a region is deemed to have an “area, then there is one and only one number which the area could possibly be. All other possibilities are “exhausted because they would violate a fundamental intuition about the nature of area, namely, that if one region lies inside another, then its area must not exceed the area of the larger region. The speaker will employ Riemann-like sums to show that if a (bounded) random variable X is to have an “expected value” $E(X)$, then there is one and only one number which $E(X)$ could possibly be. All other possibilities are “exhausted because they would violate the fundamental intuition that $X \leq Y$ should imply $E(X) \leq E(Y)$. Formula (2) becomes a theorem whose proof is similar to that of Formula (1). The formula for $E(X)$ when X has countably infinitely many values also becomes a theorem. Extensions of definitions and results to unbounded X will be discussed. Anyone not totally freaked out by Riemann sums should find this talk tolerable.

Convenor: Matthew Overs (matthew.ovens@maths.monash.edu.au).