



MONASH University

**School of Mathematical Sciences**

**Honours  
Project Topics  
Fourth Year  
2012**

(As at 13 October, 2011)

<http://www.maths.monash.edu.au/honours>

## Honours Projects 2012

The following are some of the topics available for Honours projects in 2012. The project is a very important part of your Honours year and you should choose a topic and advise the Honours co-ordinator by the end of the first week of Semester 1. Please ensure that you discuss possible topics with at least two or three possible Supervisors in February before you choose a topic. Normally, a staff member can be main Supervisor for only two Honours projects.

Some staff have not provided projects because they would prefer to discuss possible project topics with students first. You are welcome to approach all members of staff to ask about possible Honours projects in addition to those listed herein.

The project report is due at the end of second semester and includes a talk, worth 17% of the project mark. There is also a short talk at the beginning of second semester, which is not assessed. Its purpose is to help students review relevant literature clarify the goals for their project, and to obtain feedback before writing up their work.

The recommended length of the project report is 40-50 pages.

|  |           |
|--|-----------|
| <b>1. ALGEBRA AND DISCRETE MATHEMATICS RESEARCH GROUP.....</b>   | <b>5</b>  |
| <b>STAFF MEMBERS - 2012:.....</b>  | <b>5</b>  |
| <b>Projects.....</b>   | <b>5</b>  |
| Discrete Models of Critical Phenomena.....   | 5         |
| Sequences and matrices in digital communication and image watermarking.....  | 6         |
| Topics in graph decompositions and combinatorial designs.....  | 6         |
| Permutation polynomials.....   | 7         |
| Permutations.....  | 7         |
| Latin squares.....   | 8         |
| Matrix permanents.....   | 8         |
| Graph factorisations.....  | 9         |
| <b>2. ANALYSIS AND GEOMETRY RESEARCH GROUP.....</b>  | <b>10</b> |
| <b>STAFF MEMBERS - 2012:.....</b>  | <b>10</b> |
| <b>Project areas.....</b>  | <b>10</b> |
| <b>Brief introduction of project areas.....</b>  | <b>10</b> |
| <b>Projects.....</b>   | <b>12</b> |
| Prescribed mean curvature surfaces.....  | 12        |
| Discretely self-similar (DSS) singularities.....   | 13        |
| Spacetime energy.....  | 13        |
| Isoperimetric surfaces in geometry.....  | 14        |
| Newtonian limit and Post-Newtonian expansions.....   | 14        |
| Renormalization group flow.....  | 15        |
| Distributing $N$ points on a surface.....  | 15        |
| The inverse mean curvature flow and applications.....  | 17        |
| <b>3. ASTROPHYSICS AND GENERAL RELATIVITY RESEARCH GROUP.....</b>  | <b>18</b> |
| <b>STAFF MEMBERS – 2012:.....</b>  | <b>18</b> |
| Astrophysics Topics (School of Physics).....   | 18        |
| <b>Projects.....</b>   | <b>19</b> |
| Constructing initial data for Brill waves.....   | 19        |
| Solving the Rubik's cube and the Rush Hour puzzle.....   | 20        |
| Maximal slicing of a Schwarzschild black hole.....   | 20        |
| Alfvénic Oscillations in Magnetic Flux Tubes.....  | 21        |
| The Photospheric Velocity Field of Active Regions Derived from SDO/HMI Data.....   | 22        |
| TeV Gamma ray absorption from binary black holes.....  | 22        |
| TeV radiation from Blazars.....  | 23        |
| In search of acoustic aurorae on the solar surface.....  | 23        |
| The multi-instrument burst archive (MINIBAR).....  | 24        |
| Burst oscillations and anisotropic x-ray emission in neutron stars.....  | 24        |
| Exploring the extreme transient sky.....   | 25        |
| Investigating non-linear effects in planet formation.....  | 25        |
| The Composition of Stars in Clusters - Tracing the Chemical History of the Galaxy.....   | 26        |
| Spectroscopy and the Composition of Stars in Globular Clusters.....  | 27        |
| Making carbon in the universe: implications for life?.....   | 27        |
| Mixing in Red Giants.....  | 28        |
| Making fluorine in stars.....  | 28        |
| Nucleosynthesis in asymptotic giant branch stars.....  | 28        |
| Nucleosynthesis during the core helium flash.....  | 29        |
| Extending polytropic models.....   | 29        |
| H burning and its uncertainties.....   | 30        |
| Core Helium Burning - too long under the carpet!.....  | 30        |
| The First Dredge-Up in Stars in Metal-Poor Globular Clusters.....  | 30        |
| Enhanced Mass-Loss on the Horizontal Branch.....   | 31        |
| Mixing at the Core Flash.....  | 31        |
| The half life of samarium-151 and the europium isotopic composition in meteoritic stardust grains and old stars.....                               | 31        |
| The production of tantalum-180, the rarest nucleus in the universe.....  | 32        |
| The production of lutetium-176 and hafnium-176.....  | 33        |
| The neon and sodium proton-capture rates, the formation of the neutron source, and the composition of carbon-enhanced metal-poor (CEMP) stars..... | 33        |
| Interpolation of the electron-capture rates of calcium-41 for inclusion in stellar modelling.....  | 34        |
| Interpolation of the electron-capture rates of lead-205 for inclusion in stellar modelling.....  | 34        |
| The last gasp: tidal squeezing of an accretion disc by merging supermassive black holes.....   | 35        |
| Sub-resolution models for supersonic turbulence.....   | 35        |
| The Bardeen-Petterson effect in black hole accretion discs.....  | 36        |
| Understanding the Hall effect in star formation.....   | 36        |

|   |           |
|---|-----------|
| Magnetic diffusion processes in star formation.....   | 36        |
| <b>4. ATMOSPHERIC SCIENCE AND GEOPHYSICAL FLUID DYNAMICS RESEARCH GROUP ...</b>                           | <b>37</b> |
| <b>STAFF MEMBERS - 2012:.....</b>   | <b>37</b> |
| <b>Projects.....</b>  | <b>37</b> |
| Organised structures in the atmospheric boundary layer .....  | 37        |
| Modelling of the Morning Glory as interacting nonlinear waves .....                                       | 38        |
| Periodic solitary wave packets.....   | 38        |
| Shear flows past a cape.....  | 39        |
| Effect of shear on internal solitary waves.....   | 39        |
| Large scale climate dynamics.....   | 40        |
| Simple climate model projects .....   | 40        |
| How stochastic are tropical thunderstorms?.....   | 41        |
| Evaluation of a stochastic convection model .....   | 41        |
| Diffusively-driven stratified flows.....  | 42        |
| The influence of sea breezes and orography on frontal passages at Cape Grim, Tasmania.....                | 43        |
| <b>5. APPLIED AND COMPUTATIONAL MATHEMATICS .....</b>   | <b>44</b> |
| <b>STAFF MEMBERS - 2012:.....</b>   | <b>44</b> |
| <b>Projects.....</b>  | <b>45</b> |
| Modelling traffic on road networks using stochastic cellular automata.....                                | 45        |
| Simulations of swimming fish .....  | 46        |
| Extending polytropic models .....   | 47        |
| Exploring the dynamics and simulation of rotating linked rigid bodies.....                                | 47        |
| The bathtub vortex.....   | 48        |
| Resolving corner singularities in viscous flow .....  | 49        |
| On the roll-up of a ‘vortex sheet’ .....  | 50        |
| The Immersed Interface Method for solving elliptic PDEs .....   | 51        |
| Sub-resolution models for supersonic turbulence .....   | 52        |
| <b>Projects in Collaboration with the Computational Fluid Dynamics (CFD) Group at CSIRO</b>               |           |
| <b>Mathematical and Information Sciences (CMIS).....</b>  | <b>52</b> |
| Effect of particle shape on the shear bands characteristics in granular materials.....                    | 53        |
| Mathematical modelling of complex fluids .....  | 53        |
| Effect of Particle Shape on Alluvial and Gravity Driven Granular Bedforms.....                            | 54        |
| Modelling void formation and jet collapse using SPH.....  | 54        |
| <b>Projects offered jointly with School of GeoSciences and School of Mathematical Sciences.....</b>       | <b>54</b> |
| The dynamics of continental crust in early Earth, on Venus and Mars - <i>Joint with Geosciences</i> ..... | 54        |
| Modelling the dynamics of subduction zones.....   | 55        |
| Analogue and numerical models of crustal deformation models.....  | 55        |
| <b>6. STATISTICAL MATHEMATICS RESEARCH GROUP .....</b>  | <b>56</b> |
| <b>STAFF MEMBERS - 2012:.....</b>   | <b>56</b> |
| <b>Projects.....</b>  | <b>56</b> |
| Fixed Interval Scheduling.....  | 56        |
| Markov-chain Monte Carlo methods in Statistical Mechanics .....   | 57        |
| Optimising in-stream nutrient uptake studies.....   | 58        |
| Financial mathematics.....  | 59        |
| Characterisations and probability distributions .....   | 59        |
| Probability, a measure theory approach.....   | 59        |
| Learning about genes using Bayesian classification .....  | 60        |
| Detecting DNA motifs that drift.....  | 60        |
| Modelling DNA evolution under biological constraint .....   | 60        |
| Associations between phenotypes and biological networks .....   | 61        |
| Identifying biological networks via graph-based clustering algorithms .....                               | 61        |
| Modelling the Brisbane fire ant invasion .....  | 61        |
| Bayesian network inference in systems biology.....  | 62        |
| Stochastic simulation and evaluation of financial market models .....                                     | 62        |
| Stochastic simulation of biochemical reaction systems.....  | 63        |
| Random walks and electrical resistance on graphs. ....  | 64        |
| Percolation .....   | 65        |
| Stochastic simulation and control of Markov chains .....  | 66        |
| The Travelling Salesman Problem and understanding what makes it hard .....                                | 66        |
| Modelling Stem Cell Decision Making.....  | 67        |
| Mathematical modelling of regulatory networks based on microarray gene expression data .....              | 67        |
| Mathematical modelling of cancer therapy.....   | 68        |

# 1. ALGEBRA AND DISCRETE MATHEMATICS RESEARCH GROUP

## Staff members - 2012:

Dr Tim Garoni (*also a member of Statistical Mathematics Research Group and Applied and Computational Mathematics Research Group*)

Dr Tom Hall

Dr Daniel Horsley

Dr Burkard Polster (*also member of Analysis and Geometry research group*) – *not available sem. 1 2012- not offering projects in 2012.*

Dr Ian Wanless

## Projects

### Discrete Models of Critical Phenomena

**Supervisor: Tim Garoni**

**Co-supervisor: Greg Markowsky**

Background: Many probabilistic models defined on graphs possess special “critical” values of their parameters, at which long-range order develops and fractals emerge. Examples of such models include percolation, in which one studies the connectivity properties of random spanning subgraphs of some fixed graph, and the Ising/Potts models in which one studies random vertex colourings. The study of such “critical phenomena” has been a fertile area in mathematical physics for three quarters of a century. A wide variety of methods are used to study these models, ranging from rigorous combinatorial, algebraic and probabilistic methods, through to large-scale computer experiments and non-rigorous methods of theoretical physics. In the case of planar graphs, the conjectured conformal invariance of certain continuum limits of these discrete models has recently given rise to some very significant advances.

Project Outline: The project would start with a review of the required basic background in mathematical physics (statistical mechanics); the two references listed below being a good place to start. From there, a range of possible projects is available, and depending on your interests the project could include combinatorial, computational, and/or probabilistic aspects to a greater or lesser degree. There would be scope for an investigation that may lead to new results, particularly if the project involves computer experiments.

#### References:

Slade, G “Probabilistic Models of Critical Phenomena” in *The Princeton Companion to Mathematics*, edited by T. Gowers, Princeton University Press (2008).

Grimmett, G “Probability on Graphs” Cambridge University Press (2010), available online free (legally) at <http://www.statslab.cam.ac.uk/~grg/books/pgs.html>.

## **Sequences and matrices in digital communication and image watermarking**

**Supervisor: Tom Hall**

**Co-Supervisor: Andrew Tirkel**

**Background:** Sequences, over finite alphabets (often 0 and 1), with balance and two-valued autocorrelation have found applications in communications and radar: a good exposition is given in books by Golomb (Shift Register Sequences) and Schroeder (Number Theory in Science and Communications).

In digital watermarking of images, important tools are matrices with good autocorrelation and cross-correlation.

**Project Outline:** These projects would continue work by Tom Hall and Andrew Tirkel, which provided a construction of a new family of matrices with good cross-correlation and good autocorrelation. As in the method of construction of some perfect maps, their method starts with a suitable seed column and generates the further columns by a sequence of cyclic shifts. Suitable seed columns include all those with good balance and autocorrelation. When the seed column has two-valued autocorrelation, the constructed matrices have three-valued auto and cross-correlation.

The students could join Andrew and Tom in examining their new idea of reversing the above process by building new, and long-sought-after, pseudonoise sequences from matrices.

### **References:**

Golomb, "Shift register sequences".

Schroeder, "Number theory in science and communications".

## **Topics in graph decompositions and combinatorial designs**

**Supervisor(s): Daniel Horsley**

**Co-supervisor: TBA**

**Background:** Combinatorial design theory is an area of mathematics that studies arrangements of objects that are in some sense "balanced". It has applications to the design of experiments, to coding and cryptography, to traffic grooming in networks, and to numerous other areas. Many problems in combinatorial design theory can be considered as problems concerning decomposing graphs into edge-disjoint subgraphs, and this way of viewing them has led to many important results and insights.

**Objectives:** This project will involve investigating one the many easily accessible problems in the area of graph decompositions and combinatorial designs. Possible problems could concern

- embedding Steiner triple systems;
- colouring graph designs;
- resolutions of Steiner triple systems;
- decompositions of graphs into cycles; or
- infinite designs.

To discuss details of possible problems contact Daniel Horsley: danhorsley@gmail.com.

**Expectations:** A reasonable degree of mathematical maturity.

**Assumed Knowledge:** None.

**Reading:** To be determined depending on the specific problem considered.

## Permutation polynomials

**Supervisor:** Ian Wanless

**Co-supervisor:** TBA

Let  $F$  be a finite field of order  $q$ . Any permutation of  $F$  can be achieved by evaluating a polynomial of degree at most  $q-2$ . However, not all polynomials produce permutations.

In this project you would study the polynomials that do. What makes the special? Which small degree polynomials work?

We would also look at so called "complete mappings" and "orthomorphisms" in finite fields. There would be scope for an investigation that may lead to new results.

### Reference:

Lidl and Niederreiter, Finite fields,  
Encyclopedia of Mathematics and its Applications 20.  
Cambridge University Press, 1997.

## Permutations

**Supervisor:** Ian Wanless

**Co-supervisor:** Graham Farr (Clayton School of IT)

Background: Let  $n$  be a positive integer and let  $S$  denote the set of all permutations of the numbers  $1, 2, 3, \dots, n$ . So if  $n=3$  then  $S=\{123, 132, 213, 231, 312, 321\}$ . Consider the following game played with these permutations.

The game has three stages, and each person sees the other person's choices:

- 1) I choose a subset  $T$  of  $S$ .
- 2) You choose a permutation  $p$  in  $S$ .
- 3) I choose a permutation  $q$  in  $T$ .

The score is then calculated as the number of positions in which  $p$  and  $q$  agree. For example, if  $p=132$  and  $q=231$  then the score is 1 since  $p$  and  $q$  agree in the second position.

My aim is to score as highly as possible and your aim is to keep the score as low as possible.

Question: Suppose I want to guarantee a score of at least  $s$ . How small can  $T$  be?

This question has been answered only in very simple cases, such as  $s=1$  or  $s=n-1$ . Even solving the  $s=2$  case might answer some important problems in discrete mathematics.

Project Outline: The game has interesting connections with graph theory, coding theory and combinatorics, which will be explored in this project. There is plenty of scope for original work on the problem.

### References:

Contact Ian Wanless, [Ian.Wanless@monash.edu](mailto:Ian.Wanless@monash.edu)

## Latin squares

**Supervisor:** Ian Wanless

**Co-Supervisor:** Burkard Polster

Background: Latin squares are a two dimensional analogue of permutations. A Latin square of order  $n$  is an  $n$  by  $n$  matrix in which each of  $n$  symbols occurs once in each row and in each column. These days you'll find Latin squares on the puzzle page of every major newspaper as well as in the schedule for sports tournaments and statistical experiments. In pure mathematics, Latin squares are fundamental objects. Every finite group is defined by its Cayley table ("multiplication" table), which is a Latin square. Similarly, in finite geometry projective planes are defined by certain sets of Latin squares.

Project Outline: There are a range of possible projects topics available, exploring the structure and uses of Latin squares. All would be combinatorial in nature but depending on your interests they could also have an algebraic, algorithmic, probabilistic, geometric or enumerative flavour.

### References:

Laywine, C.F. and Mullen, G.L. "Discrete mathematics using Latin squares", Wiley, New York, 1998.  
Recorded Lectures (especially lectures 1,3,5,6,7) at: <http://qtss.amsi.org.au/SummerSchool2004gra.html>

## Matrix permanents

**Supervisor:** Ian Wanless

**Co-Supervisor:** Burkard Polster

Background: A diagonal of an " $n$  by  $n$ " matrix  $A$  is a set of  $n$  entries with no two in the same row or same column. The permanent of  $A$  is the sum over all  $n!$  diagonals of  $A$  of the product of the entries on that diagonal. The permanent seems to be a simplified version of the determinant but in fact it is provably harder to calculate! Nevertheless, the permanent arises in many combinatorial problems (eg. you can use it to count so-called "derangements", which are permutations with no fixed points). It also arises in many unexpected places.

There are many possible projects, but one interesting one would be to look at matrices whose permanent and determinant agree, and their surprising connections with triangulations of the plane in which each triangle is coloured either black or white and the triangles on either side of any edge have different colours.

### References:

Minc, H, "Permanents", Encyclopedia Math. Appl. 6, Addison-Wesley, Reading, Mass., 1978.  
Brualdi, R.A. and Ryser, H.J., "Combinatorial matrix theory", Encyclopedia Math. Appl. 39, Cambridge University Press, 1991.  
Recorded Lectures (especially lectures 4,9) at: <http://qtss.amsi.org.au/SummerSchool2004gra.html>

## **Graph factorisations**

**Supervisor: Ian Wanless**

**Co-Supervisor: Graham Farr - Clayton School of IT**

Background: Suppose  $n$  points are placed in general position (no 4 are co-planar). Between each pair of points, colour the line joining those points subject to the conditions:

- (i) Only  $n-1$  colours are available.
- (ii) No two lines meeting at a point may have the same colour.
- (iii) For every choice of two points and two colours it must be possible to travel between the chosen points along lines of the chosen colours.

In graph theory terminology you have found a perfect 1-factorisation of the complete graph  $K_n$ .

For a project in this area you would study what is known (and what is not known) about perfect factorisations. This could include some group theory if you want to study their symmetries, or some computational work, looking for perfect factorisations (but it doesn't have to involve either).

### **References:**

Wallis, W.D., "One-factorizations", Math. Appl. 390, Kluwer Academic, Dordrecht, 1997.

## 2. ANALYSIS AND GEOMETRY RESEARCH GROUP

### Staff members - 2012:

Professor Robert Bartnik - *(also member of Applied and Computational Mathematics research group)*

Dr Bolis Basit - *(Research Associate)*

Dr Eric Chu - *(adjunct member and also adjunct member of Applied and Computational Mathematics research group)*

Dr Anthony Lun - *(also member of Astrophysics & General Relativity research group)*

Dr Todd Oliynyk

Dr Burkard Polster - *(also member of Algebra and Discrete Mathematics research group)– not available sem. 1 2012- not offering projects in 2012)*

A/Prof. Alan Pryde

Dr Gilbert Weinstein

### Project areas

Honours projects are available in the following research areas. The exact project will be decided in consultation between the relevant staff member and the Honours student.

Geometric calculus of variations and geometric measure theory (Robert Bartnik )

Geometric evolution equations (Todd Oliynyk, Gilbert Weinstein)

Mathematical relativity (Robert Bartnik, Tony Lun, Todd Oliynyk, Gilbert Weinstein)

Functional analysis and operator theory (Alan Pryde)

Theory of almost periodic functions (Bolis Basit, Alan Pryde)

Topological geometry ( Burkard Polster)

### Brief introduction of project areas

Calculus of Variations and geometric PDE

In the geometric calculus of variations, optimal configurations associated with a variety of geometric energies are studied. Examples include minimal and prescribed mean curvature surfaces which are optimal in the sense that they have smallest surface area when compared to nearby surfaces. These mathematical objects model interfaces occurring in nature such as soap films, soap bubbles, crystal surfaces, grain boundaries, capillary surfaces and surfactant interfaces. Further applications arise in general relativity where so called maximal hyper-surfaces correspond to maximal states of expansion of the universe.

## Geometric evolution equations

Geometric evolution equations are diffusion equations for geometric functions or tensors which improve the geometric structure of surfaces or other geometric objects. They often arise from steepest descent methods associated to geometric energies (see calculus of variations) and thereby provide explicit methods for finding optimal configurations. Examples include the mean curvature flow of surfaces and the Ricci flow of metrics on manifolds. In mean curvature flow, a surface is deformed at every point in the direction of the surface normal with speed given by the average curvature of the surface at that point. This provides a mathematical model for the motion of natural interfaces such as grain boundaries. Our research focuses on the structure of singularities which can form under mean curvature flow and on ways of continuing the evolution process beyond such singularities. We also consider flows with additional constraints such as area and volume constraints and with fixed and free boundary conditions. The volume preserving flow provides a mathematical way of creating constant mean curvature (soap bubble) surfaces. Mathematical techniques include differential geometry, non-linear partial differential equations, geometric measure theory and more recently matched asymptotic expansion which are also often used in applied mathematics.

## Mathematical General Relativity

Mathematical General Relativity studies Einstein's equations of gravitation; solutions of these equations determine the geometry and curvature of space-time. Phenomena such as black holes, gravitational radiation, spacetime singularities and cosmic censorship lead to many interesting mathematical questions. These questions are of great importance in astrophysics and theoretical physics, where many similar problems arise

Spacetimes are 4 dimensional geometric objects with curvature satisfying the Einstein equations. The defining equations are geometric partial differential equations, typically of hyperbolic type, although elliptic and even parabolic equations can arise. But many of the most interesting questions, such as the inspiral collapse of a binary black hole system, or the development of singularities, cannot be answered by analysis alone, and numerical simulation is becoming an increasingly important tool.

In particular, solutions to Einstein's equations can be obtained as time evolutions of suitable initial conditions (which correspond to present states of the universe). One then studies small but physically realistic perturbations of some of the well-known solutions which correspond to empty space (Minkowski space) or to gravitational models for irrotational black holes (Schwarzschild solution). In order to solve Einstein's equations it is convenient to rewrite them with respect to natural time gauges. These gauges consist of three-dimensional hypersurfaces with geometric properties often similar to the minimal and prescribed mean curvature surfaces discussed above.

## Topological geometry

Topological geometry is the study of incidence geometries endowed with topologies that are compatible with the geometric structure. For example, a topological projective plane is a projective plane such that the two geometric operations of joining distinct points and intersecting distinct lines are continuous (with respect to given topologies on the point set and on the line set). Other important incidence geometries include circle planes, generalized polygons and buildings. As in other branches of incidence geometry, topological geometers are interested in questions of classification, characterization of the classical examples, networking between different types of geometries, etc.

# Projects

## Prescribed mean curvature surfaces

**Supervisor:**        **Robert Bartnik**

**Co-supervisor(s):** **Todd Oliynyk**

Background: Soap films and soap bubbles are examples of surfaces of constant mean curvature. The mathematical equations describing them are non-linear, second order, elliptic partial differential equations. Other examples of prescribed mean curvature are capillary surfaces and also maximal hypersurfaces in relativity. There is extensive literature, covering different mathematical approaches for proving existence, regularity and obtaining information on the shapes of such surfaces.

### Possible projects:

Isoperimetric property of the sphere

Delaunay surfaces (classification of axially symmetric constant mean curvature surfaces)

Existence of solutions (either by classical partial differential equations methods, or measure theoretical ones)

Gradient estimates (classical partial differential methods)

Construction of minimal surfaces - Weierstrass representation

Construction of minimal surfaces - geometric heat flow methods

Construction of maximal hypersurfaces in spacetime – apriori estimates: general relativity.

## **Discretely self-similar (DSS) singularities**

**Supervisor:**           **Robert Bartnik**

**Co-Supervisor:**       **Todd Oliynyk**

Background: Motivated by the pioneering analysis of the spherically symmetric Einstein equations with massless scalar field matter by Demitri Christodoulou, around 1990 Choptuik discovered numerically that spacetimes at the borderline of forming a black hole, instead developed a naked singularity with a new and totally unexpected structure.

One critical issue is to discover whether this DSS singularity is a true (non-removable) singularity or not. Numerical evidence suggests that it is removable, but this evidence is not conclusive and may be numerical artifact.

### **Possible projects:**

(numerical) solve the resulting 1+1 Einstein-massless scalar field PDE, using Choptuik's adaptive mesh refinement;

(numerical) solve the 1+1 hyperbolic PDE using Stewart's (1996) double null formulation;

(numerical) Find the underlying DSS solution to high accuracy, using Gundlach's Fourier expansion;

(analytic) Prove the existence of the DSS solution using Gundlach's technique;

(analytic) Review Christodoulou's proof of global existence for the Einstein-massless scalar field equations, and the proof that some initial data can evolve to create a black hole;

(analytic or numerical) Study similar questions for the case of massive scalar field, with or without electromagnetic charge, or for the Einstein-Yang-Mills equations

## **Spacetime energy**

**Supervisor:**       **Robert Bartnik**

**Co-Supervisor:** **Gilbert Weinstein**

The proofs of the Positive Mass Theorem by Schoen and Yau (1979) and Witten (1981) formed a watershed in the application of mathematical techniques to the Einstein Equations. The two proofs are quite different and their inter-relationship is still far from understood. The Schoen-Yau proof uses quasi-linear elliptic PDE, minimal surface theory and differential geometry; the Witten proof uses spinors and existence of solutions to the elliptic Dirac equation.

### **Possible projects:**

Review the Schoen-Yau proof of the PET and the related Positive Mass Theorem, which incorporates the linear momentum of the spacetime;

Study the use of special foliations (polar coordinates) in partial proofs of the PMT based on monotone functionals, in particular the relation between the Hawking mass and the Geroch inverse mean curvature flow;

Review properties of spinors and the Witten proof of the PMT.

(Numerical) Solutions of the initial 3D spatial geometry may be constructed by solving a parabolic equation. This project will construct such solutions numerically to study their geometry.

## **Isoperimetric surfaces in geometry**

**Supervisor:** Todd Oliynyk

**Co-supervisor:** Gilbert Weinstein

Background: Isoperimetric surfaces are surfaces that minimize area subject to a volume constraint. They arise naturally in calculus of variation and provide a natural tool to attack some basic geometric problems.

### **Possible project topics under this area include:**

Review Hubert Bray's approach to Volume Comparison using isoperimetric surfaces techniques. Investigate potential applications in the study of manifolds with boundary. Review Gerhard Huisken's approach to the definition of quasi-local mass via isoperimetric surfaces in general relativity.

### **References:**

See Gilbert Weinstein

## **Newtonian limit and Post-Newtonian expansions**

**Supervisor:** Todd Oliynyk

**Co-supervisor:** TBA

Background: The Newtonian limit is the study of solutions to Einstein gravity coupled to matter in the limit that  $v/c \rightarrow 0$ , where  $v$  is a characteristic velocity scale associated to the gravitating matter and  $c$  is the speed of light. In this limit, one expects that solutions of general relativity approach solutions of Newtonian gravity in some sense. Starting from a fully relativistic solution with a well defined Newtonian limit, one can try and expand the solution in powers of  $v/c$ . The resulting expansion is known as the Post-Newtonian expansion. This produces a sequence of equations beginning with the Newtonian gravitational one. These equations can be solved to yield an approximation to the fully relativistic solution to a certain order in  $v/c$  for  $v/c$  sufficiently small.

Possible project topics under this area include:

(numerical) In general, there will be a critical time  $T_c$  after which the Newtonian solution or more generally the Post-Newtonian expansions will no longer be a valid approximation to the fully relativistic solution. The aim of this project would be to solve the spherically symmetric Einstein equations coupled to a perfect fluid and try to identify the critical time  $T_c$  for specific classes of initial data.

(analytical) In the physics literature, Post-Newtonian expansions are computed using formal expansions without any rigorous justification. Recently, I have, using PDE techniques, established the validity of the Post-Newtonian expansions in the so called near zone. The goal of this project would be to try and understand the relationship between the formal expansions used in the physics literature and the rigorous expansions I obtained using PDE techniques.

### **References:**

See Todd Oliynyk

## Renormalization group flow

**Supervisor:** Todd Oliynyk

**Co-supervisor:** Gilbert Weinstein

Background: The Renormalization Group (RG) flow arise from demanding cut-off independence of classical field theory quantization. For the special case of nonlinear sigma models, the RG equations correspond to geometrical flow equations for a Riemannian metric on a manifold. In general, the RG equations are extremely complicated. However, they do depend a small parameter and are often studied by expanding in the parameter and truncating at a certain (loop) order.

### Possible project topics under this area include:

(analytical) Ricci flow appears as the approximation to RG flow at the one-loop level for the non-linear sigma model. The aim of this project would be to carefully understand this relationship between RG and Ricci flow.

(analytical) Entropies are important quantities for understanding the behaviour of RG flow. The goal of this project would be to interpret the Perelman's entropy for Ricci flow in terms of an entropy for the full RG flow.

(numerical) In spherical symmetry, solve numerically both the first and second order RG equations. The goal of this project would be to identify regions in space-time where the first and second order RG equations are qualitatively the same and also where they differ significantly.

(analytical) The aim of this project would be to extend existing work on spherically symmetric Ricci flow to prove either global existence or singularity formulation of solutions to the second order RG equations.

### References:

See Todd Oliynyk

## Distributing $N$ points on a surface

**Supervisor:** Gilbert Weinstein

**Co-supervisor:** To be Advised

**Background:** The problem of *evenly distributing*  $N$  points on a surface, in addition to being a fascinating mathematical problem, has found numerous applications in biology, chemistry, and physics [1]. Here 'evenly distributed' can have one of several meanings. A simple interpretation is to maximize the minimal distance to the closest neighbor. On the sphere it is referred to as *Tammes' problem* or the *hard-spheres problem*. For large  $N$ , Rakhmanov, Saff and Zhou [2] have found a very efficient algorithm which produces a near optimal solution by placing points on a spiral. In [3], this was generalized to include other surfaces of revolution, such as oblate and prolate ellipsoids, and tori.

### Possible Objectives:

1. Investigate whether this algorithm can be generalized further to surfaces which are only nearly axially symmetric, or not symmetric at all.
2. Investigate whether this algorithm also gives a near optimal configuration for other energies besides the hard-spheres.
3. Attempt to give an explanation why the spiral gives a near optimal configuration.
4. Search for efficient algorithms for moderate  $N$  (e.g. 50-500).

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**Assumed Knowledge:**

**MTH3110** Differential Geometry

**MTH3/2051** Introduction to Computational Mathematics (or equivalent programming experience).

**Reading:**

E. B. SAFF AND A. B. J. KUIJLAARS, “Distributing many points on a sphere”, *The Mathematical Intelligencer* **19** Number 1 (1997), 5–11.

E. A. RAKHMANOV, E. B. SAFF, AND Y. M. ZHOU, “Minimal discrete energy on the sphere”, *Mathematical Research Letters* **1** (1994), 647–662.

E. ASKELSON AND G. WEINSTEIN, “Distributing many points on surfaces of revolution”, in preparation.

## **The inverse mean curvature flow and applications**

**Supervisor:** Gilbert Weinstein

**Co-supervisor:** To be Advised

**Background:** The inverse mean curvature flow moves each point  $p$  of a surface  $\Sigma$  in a 3-d Riemannian manifold  $M$  by the reciprocal of the mean curvature of  $\Sigma$  at  $p$ . It has been used by Huisken and Ilmanen [3] to prove a special case of the Riemannian Penrose Inequality, by Bray and Neves [2] to compute the Yamabe invariant of  $\mathbb{R}P^3$ , and more recently by Bray and Miao [1] to give an estimate on the capacity of a surface  $\Sigma$  in terms of the Willmore functional of  $\Sigma$

### **Possible Objectives:**

1. Review of the literature on Inverse Mean Curvature Flow and its applications.
2. Investigate possible generalizations.

### **Assumed Knowledge:**

**MTH3110** Differential Geometry

**MTH3011** Partial Differential Equations

### **Reading:**

H. BRAY AND P. MIAO “On the capacity of surfaces in manifolds with nonnegative scalar curvature”, *Invent. Maths.* 172 (2008), no. 3, 459-475

H. BRAY AND A. NEVES, “Classification of prime 3-manifolds with Yamabe invariant greater than  $\mathbb{R}P^3$ ”, *Ann. of Math. (2)* 159 (2004), no. 1, 407-424

T. HUISKEN AND T. ILMANEN, “The inverse mean curvature flow and the Riemannian Penrose inequality”, *J. Differential Geom.* 59 (2001), no. 3, 353-437.

### 3. ASTROPHYSICS AND GENERAL RELATIVITY RESEARCH GROUP

#### Staff members – 2012:

Dr Leo Brewin - *(also member of Applied and Computational Mathematics research group)*

Professor Paul Cally - *(also member of Applied and Computational Mathematics research group)*

Dr Simon Campbell

Dr Alina.-C. Donea

Dr Duncan Galloway - *(joint appointment with School of Physics)*

Dr Guillaume Laibe

Professor John Lattanzio - *(also member of Applied and Computational Mathematics research group)*

Dr Maria Lugaro

Dr Tony Lun - *(also member of Analysis and Geometry research group)*

Dr Rosemary Mardling - *(also member of Applied and Computational Mathematics research group)*

Professor Joe Monaghan – *(also member of both Applied & Computational Mathematics and Geophysical Fluid Dynamics research groups)*

Dr Daniel Price - *(also member of Applied and Computational Mathematics research group)*

#### **Astrophysics Topics (School of Physics)**

Please note Honours students enrolled in Astrophysics in the School Mathematical Sciences, in addition to the large array of Astrophysics Honours projects offered in the following section of this handbook, may also choose projects from a large range of Honours Projects in Astrophysics and Astronomy offered by the School of Physics, see: <http://physics.monash.edu/undergrad/honours.html> for Physics Honours Project Handbook. If you are interested in an honours project from Physics, please contact the School of Mathematical Sciences, Astrophysics Honours Convenor, Dr Alina Donea [Alina.Donea@monash.edu](mailto:Alina.Donea@monash.edu) : +61 3 990 54488, Room 319, 3<sup>rd</sup> Floor, Maths Building 28 for approval in the first instance and then contact the School of Physics Honours Coordinator, Assoc. Professor Csaba Balazs [Csaba.Balazs@monash.edu](mailto:Csaba.Balazs@monash.edu) : +61 3 990 20328 School of Physics, to arrange project supervision.

# Projects

**Constructing initial data for Brill waves.**

**Supervisor: Leo Brewin**

**Co-supervisor: Tony Lun**

Background: Einstein's theory of gravity is described by ten coupled non-linear partial differential equations for ten metric components  $g_{\{\mu\nu\}}$ . These equations can be solved by either analytical or numerical methods but not without some difficulty -- the equations are very messy!

Project Outline: In this project we will construct time symmetric initial data for a spacetime with axial symmetry (i.e. a spacetime that is symmetric under rotations around the  $z$ -axis). This initial data will represent the geometry of a universe whose curvature is generated by a Brill wave (a pure gravitational wave). This task of constructing such initial data is a non-trivial problem as it requires the solution of a single second order non-linear elliptic pde in two dimensions. The formulation of this pde is unconventional in that we will employ a novel lattice method to record the geometry of the space. The pde will be expressed in local Riemann normal coordinates (closely related to the freely falling frames often used in General Relativity).

The numerical methods required to solve the pde will be non-trivial and will range from simple iterative schemes through to non-linear multigrid methods. Unfortunately we do not have any analytic solutions to compare our answers to but there are many numerical results.

This project will introduce you to active research in numerical relativity. You will need to be comfortable with sophisticated numerical methods as well as classical general relativity and differential geometry. This project is intended for those who would like to continue studies in numerical relativity at the postgraduate level.

## References

- Eppley, Kenneth, "Evolution of time-symmetric gravitational waves: Initial data and apparent horizons". Phys.Rev.D. Vol.16 (1977) pp.1609-1614
- Brewin, Leo, "Riemann Normal Coordinates, Smooth Lattices and Numerical Relativity. Classical and Quantum Gravity". Vol.15 (1998) pp.3085-3120
- Brewin, Leo, "Long term stable integration of a maximally sliced Schwarzschild black hole using a smooth lattice method". Classical and Quantum Gravity. Vol.19(2002) pp.429-456

## **Solving the Rubik's cube and the Rush Hour puzzle.**

**Supervisor: Leo Brewin**

**Co-supervisor: Eric Chu**

Background: We all know how frustrating the Rubik's cube can be and it is tempting to either hit it with a hammer or use a published algorithm to solve the cube (though it is much more rewarding to find your own solution). Another fun puzzle is the so called Rush Hour puzzle where you slide blocks (that represent cars), horizontally and vertically, over a grid (the grid lock of cars) to free your car from the grid lock.

Project Outline: Problems of this sort can be solved by brute force where every possible move is tried and eventually a solution is found. This is a dumb way to proceed and is not something you should feel proud about (not to mention that such strategies may take far too long to be practical). A better approach uses statistical sampling to efficiently search through the set of all possible moves to find a solution. In this project we will look at two popular methods, genetic algorithms and simulated annealing. Both methods draw upon paradigms from other areas of science where the dynamics of certain physical processes (e.g. evolution and the formation of crystals) can be described as an exercise in optimization (e.g. survival of the fittest, minimum energy principles).

We will use both genetic algorithms and simulated annealing to solve both the Rubik's cube and the Rush Hour puzzles. The project will involve a mix of computer programming (in any language you prefer) and standard mathematical tools. The creative element in this project will be how you apply the methods to these problems.

### **Reference**

See Leo Brewin.

## **Maximal slicing of a Schwarzschild black hole.**

**Supervisor: Leo Brewin**

**Co-supervisor: Tony Lun**

Background: The Schwarzschild black hole is the simplest solution (beyond flat space) of the vacuum Einstein field equations. It can be used to describe the gravitational field outside a static, non-rotating spherical object. A common way to study this (and many other spacetimes) is to follow the evolution of a 3-dimensional spacelike hypersurface in a 4-dimensional spacetime. This is the so-called 3+1 split of space and time. This can be viewed either as a single 3-dimensional hypersurface on which the 3-metric evolves with time or as a sequence of separate 3-dimensional hypersurfaces stacked on top of each other (without intersections) so as to fill a portion of (or if we are lucky all of) the 4-dimensional space time. In this later approach, the collection of slices is known as a foliation of space time.

Project Outline: Beig and Ó Murchadha have studied a particular foliation known as maximal slicing in which the local volume element (of a small 3-dimensional cell) remains constant when propagated along the normals to each slice of the foliation (i.e. the trace of the second fundamental form is zero for all time). They showed that such slices converge to a limit slice (i.e. maximal slices do not foliate the whole of the spacetime) and they also presented integral expressions by which these slices can be constructed. However the integrals require great care when being evaluated numerically (they are improper elliptic integrals). The point of this project is to study the paper by Beig and Ó Murchadha and to write a computer program that can accurately evaluate the integrals for slices close to the limit slice.

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This project will introduce you to active research in numerical relativity. You will need to be comfortable with sophisticated numerical methods as well as classical general relativity and differential geometry. This project is intended for those who would like to continue studies in numerical relativity at the post-graduate level.

## References

Beig, R and Ó Murchadha, N, “Late time behaviour of the maximal slicing of the Schwarzschild black hole” Phys.Rev.D. Vol.57 (1998) pp.4728 – 4737.

## Alfvénic Oscillations in Magnetic Flux Tubes

**Supervisor: Paul Cally**

**Co-supervisor: Hamed Moradi**

Background: Alfvén waves in magnetized ionized plasmas are transverse waves driven by magnetic tension. Observations using the Solar Optical Telescope (SOT) aboard *Hinode* (De Pontieu et al 2007) and the Coronal Multi-Channel Polarimeter (CoMP) (Tomczyk et al 2007) unambiguously reveal ubiquitous Alfvénic oscillations in the solar corona, with implications both for the Sun's atmosphere and the solar wind. However, Van Doorselaere et al. (2008) argue that these are not Alfvén waves but rather kink waves, which are related but distinct waves that may exist on discrete magnetic flux tubes. Recently, Pascoe et al (2010,2011) modelled kink waves in such tubes with continuous transverse structure and found that they are rapidly resonantly absorbed on the tube boundaries, effectively becoming Alfvén waves on these boundaries.

Project outline: The student will repeat the computations of Pascoe et al, writing a code in *Mathematica*. S/he will extend the investigation by allowing density stratification along the tube, and by exploring different initial excitations.

## References:

De Pontieu, B., McIntosh, S. W., Carlsson, M., Hansteen, V. H., Tarbell, T. D., Schrijver, C. J., Title, A. M., Shine, R. A., Tsuneta, S., Katsukawa, Y., Ichimoto, K., Suematsu, Y., Shimizu, T., & Nagata, S. 2007, *Science*, 318, 1574

Pascoe, D. J., Wright, A. N., & De Moortel, I. 2010, *ApJ*, 711, 990

Pascoe, D. J., Wright, A. N., & De Moortel, I. 2011, *ApJ*, 731, 73

Tomczyk, S., McIntosh, S. W., Keil, S. L., Judge, P. G., Schad, T., Seeley, D. H., & Edmondson, J. 2007, *Science*, 317, 1192

Van Doorselaere, T., Nakariakov, V. M., & Verwichte, E. 2008, *ApJ*, 676, L73

## **The Photospheric Velocity Field of Active Regions Derived from SDO/HMI Data**

**Supervisor:** Alina Donea

**Co-supervisor:** Paul Cally

Solar active regions are observed with the Helioseismic and Magnetic Imager (HMI) on the Solar Dynamics Observatory (SDO). Since HMI continuously observes the complete solar disk at a good spatial resolution we are able to follow active regions over several days, providing the opportunity to study the evolution of the flow field during the various stages of development of the active regions. HMI measures the photospheric vector magnetic field at a cadence of 12 min. The intensity continuum maps are taken at a cadence of 45 seconds.

To determine the velocity field we use the optical flow technique DAVE4VM (differential affine velocity estimator for vector magnetograms) and apply it to the vector magnetic field data from SDO/HMI. We will learn about this technique and its use. This project gives the student the general view of the state-of-art solar data from the satellite and how to interpret the observations.

The project is new, the student and the lecturer will work step by step, together, to discover the use of the DAVE4VM code.

### **References:**

[http://ccmc.gsfc.nasa.gov/lwsrepository/DAVE4VM\\_description.php](http://ccmc.gsfc.nasa.gov/lwsrepository/DAVE4VM_description.php)

<http://wwwppd.nrl.navy.mil/whatsnew/dave/index.html>

## **TeV Gamma ray absorption from binary black holes**

**Supervisor:** Alina Donea

**Co-supervisor:** Duncan Gallway

We show that the gamma-gamma opacity of GeV and TeV photons emitted by relativistic jet, travelling in the external photon environment of a binary supermassive black hole system, cannot be neglected when studying the emission of active galaxies. A primary supermassive black hole surrounded by a standard accretion disk in symbiosis with a relativistic jet is assumed to revolve in a co-planar orbit together with a secondary black hole surrounded by a viscous Shakura-Sunyaev small accretion disk. We investigate to what extent the location of a gamma-ray emitter exposed to the anisotropic radiation field of both accretion disks affects the gamma-ray opacity.

This project is an excellent opportunity for a student to get familiar with binary systems in the universe hosting one or two holes. The project has a descriptive part, where we will identify such systems and their properties. The computational part is based on already existing codes, which the student will learn to use.

### **References:**

talk to Alina please

## **TeV radiation from Blazars**

**Supervisor:** Alina Donea

**Co-supervisor:** Duncan Galloway

We will model the very high energy radiation from blazars assuming that the relativistic jets have a light component made of electrons and positrons. We will analyse the synchrotron emission of the charged particles in a magnetized relativistic jet. Then, the synchrotron photons will be inverse Compton scattered on the energetic populations of electrons escaping the black hole. The resulting inverse Compton (IC) spectrum has a peak at very high energy gamma rays with TeV (Tera electrovolti) energies. The student will adjust the already existing codes to shape the IC spectrum accordingly to the TeV observations of blazars.

### **References:**

See Alina Donea

## **In search of acoustic aurorae on the solar surface**

**Supervisor:** Alina Donea

**Co-supervisor:** To be Advised

We are going to use a local helioseismic technique (helioseismic holography) to map acoustic images of active regions. The acoustic maps reveal the existence of extended aurorae of enhanced acoustic signal surrounding active regions. Why are they there? What mechanism produces these? Are they related to the magnetism of the region? Do they evolve? The student will use SOHO-MDI dopplergrams, magnetograms observations to map images of a significant number of active regions.

### **References:**

See Alina Donea

## **The multi-instrument burst archive (MINIBAR)**

**Supervisor: Dr Duncan Galloway**

**Co-supervisor: To be Advised**

Background: Studies of thermonuclear (X-ray) bursts in accreting neutron stars have historically relied on short observations of individual sources, resulting in (usually) a handful of bursts. Another approach is to gather large numbers of bursts from multiple sources and telescopes, and analyse the resulting combined sample to better understand the physics of this phenomena. Such an effort is currently underway at Monash with the Multi-INstrument Burst ARchive (MINIBAR), which presently consists of approximately 3500 bursts from NASA's *Rossi X-ray Timing Explorer* satellite as well as the defunct Dutch/Italian mission *BeppoSAX*. The principal efforts at present are to add bursts in public data observed by the JEM-X camera onboard ESA's *INTEGRAL* satellite, likely adding another 1000 events and with new observations continually being added. The catalog, once complete, will prove a vital resource for studies of thermonuclear bursts and will be released to the public.

Project outline: This project will involve analysis of burst data, cross calibration, and verification for addition to the burst sample. It is expected that the student will also work on burst data from newly-discovered transient neutron stars through the course of the project, and collaborate on papers resulting from this work.

This project will primarily involve analysis of reduced data from the various X-ray satellites with IDL. Opportunities exist for work with project partners at SRON (Netherlands) and DTU Space (Denmark).

## **Burst oscillations and anisotropic x-ray emission in neutron stars**

**Supervisor: Duncan Galloway**

**Co-supervisor: Dr Yuri Levin (School of Physics)**

Background: Thermonuclear (type-I) bursts are caused by unstable ignition of accumulated hydrogen and helium on the surface of neutron stars in binary systems. These events permit approximate measurement of the neutron star spin frequency via "burst oscillations", periodic variations in the X-ray intensity at frequencies characteristic for individual sources, and detected only during the bursts.

Oscillations that occur during the burst rise may be attributed to anisotropies in the surface X-ray emission as the burning spreads from the ignition point. However, oscillations may also occur after the burst peak, by which time the burning is thought to have spread over the entire surface of the star. The blackbody radius of the burst spectrum provides a measure of the physical emitting area.

Project outline: This project will compare the presence of oscillations with the measured blackbody radius using a large sample of bursts detected by NASA's *Rossi X-ray Timing Explorer* satellite, to test whether oscillations during the burst tail may be attributed to anisotropies in the X-ray emission.

This project will provide a good introduction to high-energy astrophysics, as well as expertise in analysis of X-ray data from RXTE, and possibly other satellites. Opportunities exist for collaborative work with project partners at MIT (USA).

**References:** [1] Strohmayer & Bildsten, astro-ph/0301544

## **Exploring the extreme transient sky**

**Supervisor: Duncan Galloway**

**Co-Supervisor: To be Advised**

The European Space Agency's (ESA) XMM-Newton spacecraft provides X-ray detectors featuring large effective area in the 0.2–10 keV energy range, permitting sensitive observations for faint sources at good spatial and spectral resolution. These capabilities permit detection of the brightest and faintest of X-ray sources, including accreting black holes and neutron stars, active galactic nuclei, and even distant galaxy clusters. XMM-Newton has been observing the X-ray sky for over 11 years, and has accumulated an extensive database of observations covering a substantial fraction of the sky. The 2009 release of the XMM-Newton Serendipitous Source Catalogue, the largest X-ray source catalogue ever produced, includes detections of 263,230 unique individual sources from 4,953 observations.

A substantial fraction of these sources are transients, for which the nature and characteristics are unknown. This project will involve developing strategies to select sources from the catalog and, using archived observations in other wavelengths, to deduce their nature. Work will be carried out in collaboration with researchers at the University of Sydney. The transient fraction of the catalog has barely been explored so far, so there is a good chance of discovery of new sources with unusual properties. Prompt transient followup strategies may also be adapted for use in the Variable and Slow Transients (VAST) program on the Australian Square Kilometre Array Pathfinder (ASKAP), currently under construction.

## **Investigating non-linear effects in planet formation**

**Supervisor: Guillaume Laibe**

**Co-supervisor: Daniel Price**

Background: Although over 490 planets around other stars have already been discovered, the problem of their formation remains one of the most burning questions of contemporary science. A key point is that current analytical theories are restrictive, as they do not predict the non-linear behavior of the systems considered.

Project Outline: In this project, we shall derive criteria for the development of instabilities in dusty discs, relevant to the early stages of planet formation. The project will be divided in three parts:

- 1- an analytical derivation.
- 2- writing a small piece of code in Mathematica/Maple or equivalent.
- 3- using this tool to determine how non-linear effects limit (or do not limit) the formation of planet embryos.

During this project, you will deal with the rich domains of non-linear physics and planet formation and thus learn methods applicable to a large range of scientific domains. Successful completion of the project could result in the publication of an article in an international scientific journal.

Good analytical skills, taste for burning challenges and enthusiasm are required!

## **References**

See Guillaume Laibe, email: [guillaume.laibe@monash.edu](mailto:guillaume.laibe@monash.edu)

## **The Composition of Stars in Clusters - Tracing the Chemical History of the Galaxy**

**Supervisor: Prof John Lattanzio (Monash)**

**Co-Supervisors: Dr Gayandhi de Silva (AAO) and Prof Brad Gibson (Univ Central Lancashire, UK, but visiting Monash in 2012)**

Background: Most stars are born in clusters. The oldest and most populous are the globular clusters which orbit our Galaxy. Within the galaxy itself are younger, less popular "open clusters". Recent studies have shown that the stars in globular clusters show chemical element abundance patterns that are unique to the clusters. We do not know why they are not seen in the Galaxy, but only within the globular clusters. Is it possible for similar patterns to appear in open clusters? Preliminary observations suggest that they do not, and that the stars within a given open cluster all have the same abundances. But how does the composition of the open clusters as a population compare to the patterns seen in the globular clusters? Are the open clusters showing a similar chemical enrichment history or a different history?

Project: It is proposed to collect data from the literature on the abundances in various open clusters and to compare these to the patterns seen in globular clusters. Do the open clusters show the sort of enrichment history that is seen in the Galaxy overall? Or do they share some of the patterns seen in globular clusters? The project will involve understanding stellar nucleosynthesis as well as the chemical evolution of the Galaxy, and work with a computer codes that calculate these properties.

This project will involve travel to the Australian Astronomical Observatory (in Sydney) to visit and work with Dr de Silva.

### **References:**

- De Silva, G. M.; Gibson, B. K.; Lattanzio, J.; Asplund, M., 2009, "O and Na abundance patterns in open clusters of the Galactic disk", *Astron. Astrophys.*, 500, 25.
- De Silva, G. M.; Freeman, K. C.; Bland-Hawthorn, J., 2009, "Reconstructing Fossil Substructures of the Galactic Disk: Clues from Abundance Patterns of Old Open Clusters and Moving Groups", *Publications of the Astron Soc of Australia*, 26, 11
- De Silva, G. M.; Freeman, K. C.; Asplund, M.; Bland-Hawthorn, J.; Bessell, M. S.; Collet, R., 2007, "Chemical Homogeneity in Collinder 261 and Implications for Chemical Tagging", *Astronomical Journal*, 133, 1161.

## **Spectroscopy and the Composition of Stars in Globular Clusters**

**Supervisor:** Prof John Lattanzio (Monash)

**Co-supervisor:** Dr Gayandhi de Silva (AAO)

Background: Globular clusters are the oldest and most populous stellar aggregates in existence. Recent studies have shown that the stars in globular clusters show abundance patterns that are unique to the clusters. We do not know why they are not seen in the Galaxy, but only within the globular clusters. They may even be the remnants of collisions between dwarf Galaxies and our Milky Way. A fuller understanding requires us to determine the abundances of many stars in many clusters and to compare with theoretical models so we can see what stars produced the existing patterns.

Project: We will source original data from the world's largest telescopes and then analyse this to determine the abundances of key species in globular cluster stars: perhaps Li, C, N, O, Mg, Al, Fe as well as the heavy elements made by neutron capture, such as Sr, Y, Zr, Ba, La. Stellar models that can produce these species will be compared with the abundances we measure.

This project will involve travel to the Australian Astronomical Observatory (in Sydney) to visit and work with Dr de Silva. There is also the opportunity to visit the 4m Anglo-Australian Telescope in Coonabarabran, NSW.

## **Making carbon in the universe: implications for life?**

**Supervisor:** John Lattanzio

**Co-Supervisor:** To be Advised

Background: This project will look at the production of carbon by red-giants. The aim will be to take the results from detailed stellar evolution calculations and include these in a new code which simulates the evolution of an entire Galaxy. With some simple approximations, we can produce a "population synthesis" code which can model a large population of stars. Then we can investigate the effect of binary stars, mass-transfer from one to another, the effect of mass-loss etc etc. It is possible, if time permits, to also include other species so we can look at the production of Oxygen also. This has many implications for the appearance of life in the Universe - planets forming in a carbon-rich environment are very different to those forming in a oxygen-rich environment!

Outline: This would be a good project for someone who wanted to improve their skills at computer programming: it will start with a very simple code to which you will add more and more and build it into a substantial piece of work.

## **References:**

See John Lattanzio.

## Mixing in Red Giants

**Supervisor:** John Lattanzio

**Co-Supervisor:** To be Advised

We will investigate the implications some extra-mixing in red-giant stars. How does this affect the surface values of carbon isotopes? Does it affect other species? What do the observations require? How deep is the mixing? What nuclear processes are enabled as a result of the mixing?

### References:

See John Lattanzio .

## Making fluorine in stars

**Supervisor:** John Lattanzio

**Co-Supervisors:** Maria Lugaro

Red-giants are the only stars known to produce fluorine. Yet the amount they make is very sensitive to details in the calculations and some uncertain reaction rates. In this project you will run existing computer codes, with different reaction rates and choices for mixing, to explore the sort of predictions made by the models.

### References:

See John Lattanzio and Maria Lugaro

## Nucleosynthesis in asymptotic giant branch stars

**Supervisor:** John Lattanzio

**Co-supervisors:** To be Advised

Background: Information about the nucleosynthetic processes that occur in asymptotic giant branch (AGB) stars is obtained both from spectroscopic observations of stars and from the measurement of isotopic ratios in pre-solar grains. Standard stellar models involving only convective mixing cannot explain the observed abundance patterns and isotopic ratios: an additional mixing process must be at work. This project will look at constraining the nature of this additional mixing process.

Outline: In this project you will use a stellar evolution code together with a postprocessing code to compute the nucleosynthesis that occurs in AGB stars. These results will be compared to what we observe. You will then look at physical mechanisms that may lead to additional mixing and add these to the postprocessing code to see how they affect the nucleosynthesis.

### References:

See John Lattanzio.

## **Nucleosynthesis during the core helium flash**

**Supervisor: John Lattanzio**

**Co-Supervisor: Joe Monaghan**

Background: After a star exhausts its central H supply it becomes a red-giant. For low mass stars this He core becomes electron-degenerate and continues to heat. When the temperature reaches about 100 million degrees we find helium burning is ignited. But due to the degenerate equation of state, this is almost explosive. The star generates about one billion times the energy of our Sun, but just for a few days. The structure of the star changes dramatically, of course. Although the details are very complicated, we know that most stars survive this phase, as we see them in the next stage of their lives.

Project Outline: Standard calculations with the standard assumptions (hydrostatic equilibrium, instantaneous convective mixing) seem to get through this phase, although clearly the details require multi-dimensional hydrodynamical calculations. Nevertheless, since models made with these assumptions match the later phases, these standard assumptions must be OK for most stars. In this project you will evolve stars through this phase using a stellar evolution code, investigating the uncertainties in the calculations. Further, a separate code calculates the detailed nuclear reactions occurring and we will investigate the reactions and possible mixing mechanisms. Are there observational tests we can apply to the models?

### **References:**

See John Lattanzio.

## **Extending polytropic models**

**Supervisor: John Lattanzio**

**Co-Supervisor: Paul Cally**

Background: Simple polytropic models have many uses in stellar astrophysics. The simple approach used in 3rd year can be extended by adding an equation for the luminosity as well. In this case one can include the H burning reactions and calculate a luminosity for the model.

Project Outline: The idea is to try to construct polytropic models that represent the main phases of stellar evolution around the HR diagram. How can we best simulate a main sequence? Can we simply use the pp and cno cycle reactions? Will it be necessary to make some modifications? eg are all ms stars equally well approximated by polytropes with  $n=3$ ? Maybe we will need to vary  $n$ ? Can we simulate a red-giant somehow? Further, what is the best way to solve the Lane-Emden equation? We will investigate using a Runge-Kutta-Fehlberg scheme with a maximum specified error at each step. We can also write the Lane-Emden equation as a DE for  $\xi$  as a function of theta, so that the boundaries are now well known!

### **References:**

See third year ASP3012 notes and John Lattanzio.

## **H burning and its uncertainties**

**Supervisor: John Lattanzio**

**Co-Supervisor: TBA**

Background: What are the conditions for hydrogen burning? How do the results of H burning depend on the temperature and the density of the burning environment?

Project Outline: In this project you will write a code to solve the nuclear burning reactions for H by pp chains and CNO cycles. Then when that is running we can add the less energetically important Ne-Na and Mg-Al chains, which are used as observational diagnostics of the burning region.

### **References:**

see Prof John Lattanzio

## **Core Helium Burning - too long under the carpet!**

**Supervisor: John Lattanzio**

**Co-Supervisor: To Be Advised**

Background: One of the most controversial stages of evolution of low mass stars is the end of the core helium burning phase. There is an instability that is predicted by the models - but the details are not clear. Is it a real instability? Yes, if you assume instant mixing. But mixing is not instant.

Project Outline: Can we learn something by using a code that allows for time-dependent mixing processes?

### **References:**

see Prof John Lattanzio

## **The First Dredge-Up in Stars in Metal-Poor Globular Clusters**

**Supervisor: John Lattanzio**

**Co-Supervisor: Simon Campbell**

Background: Globular Clusters are among the oldest objects in the Universe. The stars in them show abundance patterns that are not seen in any other known stars! Various models have been proposed to explain this with varying degrees of success. But recently we have discovered that the most metal-poor globular clusters seem to show that the stars are behaving in a way that is not consistent with our understanding of the first dredge-up event on the giant branch. The stars seem to be mixing down to depths much deeper than predicted. In this project we will investigate the depth of mixing and how it varies with mass and composition in the range appropriate to these stars. We will investigate possible ways to make the mixing deeper.

### **References:**

see Prof John Lattanzio

## **Enhanced Mass-Loss on the Horizontal Branch**

**Supervisor: John Lattanzio**

**Co-Supervisor: Simon Campbell**

Background: After stars ignite helium at the tip of the giant branch, they settle down to 100 million years of burning helium in their core. Some recent observations indicate that these stars are experiencing a higher rate of mass-loss than is expected. This can cause the star to alter its evolution and these stars may not continue to the AGB phase. Recent work by Simon Campbell indicates that some globular cluster stars do not reach the AGB? Could this be related? In this project we will make models of stars with enhanced mass-loss during their helium burning evolution and see what effect this has on the distribution of stars in globular clusters.

### **References:**

see Prof John Lattanzio

## **Mixing at the Core Flash**

**Supervisor: John Lattanzio**

**Co-Supervisor: Simon Campbell**

When a red-giant reaches the top of the giant branch, the temperatures in the core are high enough for He burning to begin. In a star with a degenerate core, this ignition is explosive. Some recent observations seem to indicate that mixing may be initiated that brings the products of the explosive burning to the surface. Some RR Lyrae variables (post-core flash objects) are rich in Carbon (produced at the flash). Some other post-core-flash stars seem rich in Lithium. Can we make lithium at the flash? In this project we will investigate explosive mixing in red-giants and see if we can mimic a situation that might explain these observations.

### **References:**

see Prof John Lattanzio

## **The half life of samarium-151 and the europium isotopic composition in meteoritic stardust grains and old stars**

**Supervisor: Maria Lugaro**

**Co-Supervisor: To be Advised**

Samarium-151 ( $^{151}\text{Sm}$ ) is a radioactive nucleus with a halflife dependent on the temperature. During neutron fluxes, the relative importance of its decay rate and its neutron-capture rate controls the production of the two stable isotopes of europium,  $\text{Eu}151$  and  $\text{Eu}153$ . The ratio of these two isotopes has been measured in meteoritic stardust silicon carbide ( $\text{SiC}$ ) grains, which originated in asymptotic giant branch (AGB) stars where neutron captures occur and in a few carbon-enhanced metal-poor stars (CEMP) showing the signature of mass transfer from a former AGB stellar companion. This project involves including the temperature-dependence of the decay rate of  $^{151}\text{Sm}$  in a code that models nucleosynthesis in AGB and then running stellar models to test if it is possible to reproduce the  $\text{Eu}151/\text{Eu}153$  ratio measured in stardust  $\text{SiC}$  grains and in CEMP stars.

This project is performed in collaboration with colleagues at the Research School of Earth Sciences at the Australian National University.

**References:**

"Europium Isotope Ratios in s-Process Element-enhanced Metal-poor Stars: A New Probe of the  $^{151}\text{Sm}$  Branching" Aoki, W. et al. 2003, *The Astrophysical Journal*, Volume 592, Issue 2, pp. L67-L70

"Eu isotope measurements on single SiC grains from the Murchison meteorite: A new probe of s-process conditions in parent Asymptotic Giant Branch stars" Terada, K. et al. 2006 *New Astronomy Reviews*, Volume 50, Issue 7-8, p. 582-586

**The production of tantalum-180, the rarest nucleus in the universe**

**Supervisor: Maria Lugaro**

**Co-Supervisor: To be Advised**

Tantalum (Ta) has the lowest cosmic abundance of all elements, and its isotope Ta180 is the rarest stable nucleus of all. This is because Ta180 is difficult to produce as it lies on the proton-rich side of the main neutron-capture process path, but it is shielded from proton-capture production by tungsten-180. However, if the temperature is high enough in the He burning shell of asymptotic giant branch (AGB) stars the normally stable nucleus hafnium-179 can decay into Ta179. In the presence of neutrons, neutron captures on Ta179 produce both the ground unstable state of Ta180 and the isomeric stable state of Ta180. The two states can then interchange with each other eventually determining the final abundance of Ta180 that can be produced by this process. This project involves implementing the different nuclear reactions and interactions that are needed to describe the production of Ta180 in AGB stars and run stellar models to quantify the final result.

This project is performed in collaboration with colleagues at the Department of Nuclear Physics at the Australian National University.

**References:**

"Stellar neutron capture on  $^{180}\text{Tm}$ . II. Defining the s -process contribution to nature's rarest isotope" Kaeppeler F. et al. 2004 *Physical Review C*, vol. 69, Issue 5, id. 055802

## **The production of lutetium-176 and hafnium-176**

**Supervisor: Maria Lugaro**

**Co-Supervisor: To be Advised**

Lutetium-176 (Lu176) and hafnium-176 (Hf176) are nuclei that can only be produced by the slow neutron-capture (s-)process that occurs in the helium intershell of asymptotic giant branch (AGB) stars. Thus, their relative abundances in the solar system have to be matched by models of the s process. However, modelling their production is made complex by the structure of the Lu176 nucleus, where, depending on the temperature, the unstable isomeric state can be coupled with the stable ground state. This project involves implementing the different nuclear reactions and interactions that are needed to describe the production of Lu176 and Hf176 in AGB stars and run stellar models to quantify the final result. Current models are not able to find a satisfactory match to the observed abundances. Our models will be able for the first time to consider the details of nuclear structure together with mixing and burning to provide a breakthrough in this complex problem.

This project is performed in collaboration with colleagues at the Department of Nuclear Physics at the Australian National University.

### **References:**

"Properties of the  $5^{-}$  state at 839 keV in Lu176 and the s-process branching at A=176" Mohr, P. et al. 2009, Physical Review C, vol. 79, Issue 4, id. 045804

## **The neon and sodium proton-capture rates, the formation of the neutron source, and the composition of carbon-enhanced metal-poor (CEMP) stars**

**Supervisor: Maria Lugaro**

**Co-Supervisor: To be Advised**

CEMP stars are some of the oldest objects in our Galaxy and their spectroscopic observations provide constraints on nucleosynthesis in stars of very low metal-content. Many of these stars show the signature of material accreted from a more massive binary companion, which evolved through the asymptotic giant branch (AGB) phase first and produced the observed carbon, fluorine, sodium, and elements heavier than iron such as barium and lead. Models of AGB stars of low metal-content can match most of the abundance observed in CEMP stars, except that too much sodium is produced. This project investigates possible answers to this problem and involves running models of AGB stars of  $\sim 2$  solar masses and  $\sim 200$  times less metals than the Sun and check (1) the effect of the uncertainties in the proton-capture rates that produce and destroy sodium (2) the effect of the details of the mixing mechanism supposed to form the neutron source in these stars.

This project is performed in collaboration with colleagues at the Research School of Astronomy and Astrophysics at the Australian National University.

### **References:**

"The s-process in low-metallicity asymptotic giant branch stars and the composition of carbon-enhanced metal-poor stars" Lugaro et al. 2011, Astrophysical Journal, submitted (please see Maria Lugaro me for a copy)

## **Interpolation of the electron-capture rates of calcium-41 for inclusion in stellar modelling**

**Supervisor: Maria Lugaro**

**Co-Supervisor: To be Advised**

The abundances of radioactive nuclei are a fundamental output of stellar nucleosynthesis models in particular as they can be compared to the abundances of these nuclei inferred to be present in the young Solar System via meteoritic data. The radioactive nucleus calcium-41 has been found to be present in the early Solar System but its abundance is difficult to calculate in stellar models because its electron-capture rate depends on both the temperature and the density of the stellar gas. This project involves interpolating tables of the electron-capture rates of this nucleus so that a value for the rate can be easily computed at any temperature and density and incorporated in stellar model. Models for asymptotic giant branch (AGB) and Super-AGB stars of masses  $\sim 6 - 10$  solar masses will then be computed and the predicted abundances will be compared to meteoritic data to check if one of these AGB stars is a good candidate to have polluted the young Solar System of these radioactive nuclei.

### **References:**

"Injection of freshly synthesized Ca-41 in the early solar nebula by an asymptotic giant branch star" Wasserburg, G. J. et al. 1995, *Astrophysical Journal*, Part 2 - Letters, vol. 440, no. 2, p. L101-L104

## **Interpolation of the electron-capture rates of lead-205 for inclusion in stellar modelling**

**Supervisor: Maria Lugaro**

**Co-Supervisor: To be Advised**

Lead-205 ( $Pb^{205}$ ) is a radioactive nucleus with a half-life dependent on temperature and density. Its half-life with respect to neutron-capture cross section affects the isotopic ratios of Pb, which have been measured in stardust silicon carbide (SiC) grains which originated from carbon-rich asymptotic giant branch (AGB) and are recovered from primitive meteorites. This project involves interpolating tables of the electron-capture rates of this nucleus so that a value for the rate can be easily computed at any temperature and density and incorporated in stellar model. Models for AGB stars will then be computed and the predicted abundances will be compared to the meteoritic stardust data.

This project is performed in collaboration with colleagues at the Research School of Earth Sciences at the Australian National University.

### **References:**

Avila, J. 2010 PhD thesis, ANU (please ask Maria Lugaro for a copy)

## **The last gasp: tidal squeezing of an accretion disc by merging supermassive black holes**

**Supervisor: Daniel Price**

**Co-Supervisor: Duncan Galloway**

Background: Perhaps the most major revolution in Astronomy since the invention of the telescope is set to occur in the next few years with the detection of gravitational wave signals. One of the prime targets for the LISA mission, set to launch in 2012, is to detect the 'last gasp' of merging supermassive black holes following the merger of their host galaxies. However, observations of the gravitational wave signal itself will be limited in their ability to pinpoint any given signal to any more than a range of candidates. Thus, the key challenge will be to find the 'electromagnetic counterparts' to these events.

Project Outline: In this project we will consider what happens if one of the black holes during the merger is surrounded by an accretion disc. The gravitational-wave driven merger of the two black holes is expected to tidally squeeze the disc and drive a very rapid accretion of gas onto the primary, giving a 'burst' of electromagnetic radiation accompanying the merger that may be detected with electromagnetic telescopes (with very high spatial resolution) and thus used to pinpoint the origin of the gravitational wave signal.

The problem has been studied previously in one dimension by Lodato et al. (2009) [1], with results consistent with the above. However there are many limitations to this approach that can only be addressed by performing full, three dimensional simulations. The goal of this project is to simulate the process using my 3D Smoothed Particle Hydrodynamics code, phantom. The project requires only small modifications to the code itself and the student will gain experience in running 'real' astrophysics calculations using modern supercomputing facilities. A successful completion of the project has the potential to lead to a publication in a major astronomical journal.

### **References**

Lodato, G., Nayakshin, S., King, A.R. and Pringle, J.E., MNRAS 398, 1392 (2009).

## **Sub-resolution models for supersonic turbulence**

**Supervisor: Daniel Price**

**Co-Supervisor: Joe Monaghan**

Turbulence is hugely important in many physical problems, including many in astrophysics. However, the turbulence in star-forming molecular clouds is quite different to laboratory turbulence because the motions far exceed the speed of sound. Thus, whilst a great many "sub-grid" models have been developed for simulating turbulence in the laboratory on a computer, it is not clear whether any of these can be applied in the supersonic regime.

In this project we will attempt to extend a sub-resolution turbulence model developed for the Smoothed Particle Hydrodynamics method [1,2] to deal with supersonic flows. We will consider some simple problems in one dimension whereby a shock interacts with a "turbulent" medium in order to demonstrate whether or not the method can be applied in this regime and whether or not it adequately captures the sub-resolution physics that occurs in turbulent flows. Extensions to the project could include the application to more realistic flows in two or three dimensions.

The project would suit a student in either astrophysics or applied maths who enjoys computing. Successful completion of the project has the potential to lead to a full research publication, so would be a good choice for a student considering further PhD research.

### **References**

Monaghan, J. J., Journal of Turbulence, Volume 5, Issue 1, pp. 012 (2004).

Monaghan, J. J., Monthly Notice of the Royal Astronomical Society, Volume 335, Issue 3, pp. 843-852 (2002)

### **The Bardeen-Petterson effect in black hole accretion discs**

**Supervisor: Daniel Price**

**Co-Supervisor: To be Advised**

Description: How are accretion discs around black holes warped by the spin of the black hole? Using D. Price's code, we will perform 3D simulations of accretion discs in orbit about rotating compact objects, such as black holes and neutron stars, and study the structure of warped discs produced by the Bardeen-Petterson effect from a spinning black hole.

**References:**

See Daniel Price

### **Understanding the Hall effect in star formation**

**Supervisor: Daniel Price**

**Co-Supervisor: To be advised**

Description: Magnetic fields can strongly affect the formation of stars during the collapse of protostellar clouds. However, assuming a simple description of the magnetic field is not enough to accurately compute their effects. One of the strangest processes is the Hall effect, where a diffusion arises that depends on the sign of the magnetic field. The Hall effect can do weird things, like induce rotation in a previously non-rotating cloud. For this project we will derive the basic equations describing the Hall effect in magnetised gas dynamics (magnetohydrodynamics, or MHD), and work towards a numerical algorithm that is capable of simulating Hall-MHD effects.

**References:**

See Daniel Price

### **Magnetic diffusion processes in star formation**

**Supervisor: Daniel Price**

**Co-Supervisor: To be Advised**

Description: How do stars form? The process involves turbulence, gravity, and gas dynamics in the presence of magnetic fields. In particular, simulations have shown that magnetic fields can almost completely prevent the formation of protoplanetary discs around newborn stars, presenting a problem for theory. To bring models into better agreement with observations we need to understand better the physical process that can destroy magnetic fields. In this project we will develop a numerical solution of the equations describing the ionisation balance in star-forming molecular gas, forming a small module that can be incorporated into supercomputer simulations of the star formation process.

**References:**

See Daniel Price

## 4. ATMOSPHERIC SCIENCE AND GEOPHYSICAL FLUID DYNAMICS RESEARCH GROUP

### Staff members - 2012:

Dr Danijel Belušić

Dr Simon Clarke - *(also member of Applied and Computational Mathematics research group)*

Dr Dietmar Dommenges

Professor Christian Jakob

Professor Joe Monaghan - *(also member of both Applied & Computational Mathematics and Astrophysics & General Relativity research groups)*

Assoc. Professor Michael Page - *(also member of Applied and Computational Mathematics research group)*

Professor Michael Reeder

Assoc. Professor Steve Siems

## Projects

### Organised structures in the atmospheric boundary layer

**Supervisor:** Dr Danijel Belušić

**Co-Supervisor:** To be advised

Background: Coherent structures are ubiquitous in turbulence, and include ramp-cliff patterns, microfronts, and other unclassified shapes. They influence the turbulent fluxes, mixing and the structure of the turbulent boundary layer in general. Laboratory turbulence studies have revealed certain characteristics and mechanisms of these structures, but their existence, dynamics and effects in the atmospheric boundary layer are much less known. Furthermore, the atmospheric boundary layer is home to a wide range of other more or less organized structures, such as gravity waves, solitons, drainage flows, etc., many of which have not been properly studied yet.

Project Outline: Multiple projects are possible. The aim is to investigate various features that appear in the atmospheric boundary layer using different sources of data, and to understand their effects and importance for the atmospheric dynamics. The project foci can range from analyses of individual cases, their dynamics, structure and behaviour, through scale analyses, innovative data analysis techniques, etc.

### References:

Antonia, R.A. et al., 1979: "Temperature ramps in the atmospheric surface layer", *Journal of the Atmospheric Sciences* 36, 99-108.

Mahrt, L., 2007, "Weak-wind mesoscale meandering in the nocturnal boundary layer", *Environmental Fluid Mechanics* 7, 331-347.

## **Modelling of the Morning Glory as interacting nonlinear waves**

**Supervisor: Simon Clarke**

**Co-Supervisor: To be advised**

Background: Recent simulations of the Morning Glory roll-cloud of Northern Australia have been undertaken at Monash. These suggest that its generation involves the resonant interaction of two sea-breezes. A very simple model of this interaction involves long, nonlinear waves propagating on the interfaces between three fluid layers of differing density. When the phase velocities of two of the possible interfacial wave modes is similar a pair of coupled Korteweg-de Vries equations result, which exhibit resonantly interacting solutions.

Project Outline: The aim of this project is to use numerical simulations to determine the applicability of the simplified coupled model to the environmental conditions which occur for the formation of the Morning Glory and to further understand the dynamics of its generation.

### **References:**

- Christie, D.R., "The morning glory of the Gulf of Carpentaria: a paradigm for nonlinear waves in the lower atmosphere", Australian Meteorological Magazine, 41, pp. 21-60, 1992.
- Goler, R. & Reeder, M., Journal of Atmos. Sci., accepted. (available from Michael Reeder, Room 460).
- Gottwald, G. & Grimshaw, R., "The formation of coherent structures in the context of blocking", Journal of Atmos. Sci., 56, pp. 3363-3678, 1998.

## **Periodic solitary wave packets**

**Supervisor: Simon Clarke**

**Co-Supervisor: To be advised**

Background: In telecommunications one proposed method of transmitting pulses in optical fibres is to use periodically varying fibres. This leads to what are known as "dispersion-managed solitons". An analogous situation can occur for waves in a stratified fluid with periodically varying bottom topography, except in this case it is the coefficient of the nonlinear term which varies rather than that of the dispersive term.

Project Outline: This projects aims to consider using numerical simulations and perturbation techniques whether stable solitary wave pulses are possible in such periodically varying waveguides.

### **References:**

- Clarke, S.R, Malomed, B.A. & Grimshaw, R., "Dispersion management in a Korteweg-de Vries system" Chaos 12, pp. 8-15, 2002.
- Grimshaw, R.H.J., "Internal solitary waves, in environmental stratified flows", ed. Grimshaw, R., Kluwer, pp. 1-28, 2001.

## **Shear flows past a cape**

**Supervisor: Simon Clarke**

**Co-Supervisor: To be advised**

**Background:** When a current such as the Gulf Stream or the East Australian Current interacts with topography vigorous eddies are typically shed, which can lead to separation of the current from the coast. This projects aims to consider the processes by which this can occur using a simple model of a piecewise linearly varying ocean current flowing past a semi-circular cape. In this model eddies can develop from large amplitude waves which form on the vorticity interface and propagate upstream and downstream away from the cape. Two methods can be used to consider the resulting flow; limiting the consideration to long waves and numerical integration.

**Project Outline:** The purpose of this project is to compare numerical integrations with analytical solutions and to investigate the evolution and interaction of eddies with topography.

### **References:**

Clarke, S.R., Johnson, E.R., "Topographically-forced long waves on a sheared coastal current Part 1 , The weakly nonlinear response", *Journal of Fluid Mechanics* 343, pp.131-151, 1997.

Clarke, S.R., Johnson, E.R., "Topographically-forced long waves on a sheared coastal current, Part 2. Finite amplitude waves". *Journal of Fluid Mechanics* 343, pp.153-168, 1997.

## **Effect of shear on internal solitary waves**

**Supervisor: Simon Clarke**

**Co-Supervisor: To be advised**

**Background:** In a linearly stratified fluid with weak velocity shear a variety of solitary waves can occur. For sufficiently large amplitude these will form breaking waves. However analytical calculations suggest that some varieties of velocity shear are always unstable and eventually will lead to wave breaking.

**Project Outline:** The aim of this project is to investigate various velocity shears using a full two-dimensional model of a stratified fluid and determine whether solitary waves are stable or unstable and what occurs as the amplitude of waves become large.

### **References:**

Clarke, S.R. & Grimshaw, R.H.J., "The effect of weak shear on finite amplitude internal waves", *Journal of Fluid Mechanics* 395, pp.125-159, 1999.

Grimshaw, R.H.J., "Internal solitary waves in environmental stratified flows", ed. Grimshaw, R, Kluwer, pp.1-28, 2001.

## **Large scale climate dynamics**

**Supervisor:** Dietmar Dommenges

**Co-Supervisor:** To be advised

My research focus is on the analysis and simulation of the large scale climate variability from seasonal (e.g. *El Nino*), to decadal, to centennial time scales. This includes the analysis and simulation of Climate Change and predictions of long term natural climate variations.

Plenty of possible projects can be formulated, which will either analyse observed or simulated data, study simple climate models, or develop simple toy-models.

### **References:**

<http://users.monash.edu.au/~dietmard>

## **Simple climate model projects**

**Supervisor:** Dietmar Dommenges

**Co-Supervisor:** To be advised

Background: I have developed a simple climate model that can simulate the global climate response to external forcing. It can compute 100,000 yrs of simulation per day on a standard PC computer. Thus it is a nice and simple tool that allows for a wide range of studies. Detailed projects with this simple climate model will be formulated together with the student, as there are simply too many different things that could be done with this model to list them all here.

### **Webpages:**

[http://users.monash.edu.au/~dietmard/content/GREB/GREB\\_model.html](http://users.monash.edu.au/~dietmard/content/GREB/GREB_model.html)

<http://users.monash.edu.au/~dietmard>

Last reviewed: 06/09/2011

## **How stochastic are tropical thunderstorms?**

**Supervisor: Christian Jakob**

**Co-supervisor: Laura Davies**

In climate models thunderstorms must be included as conceptual models often referred to as parametrizations. For those to work there has to be a relationship between what the model equations resolve and the thunderstorms themselves. In our current research we have shown that this relationship is partially deterministic and partially stochastic. This project will investigate how the degree of stochasticity depends on the size of the grid-box in which the analysis is performed.

### **References:**

<http://users.monash.edu.au/~cjacob/>

## **Evaluation of a stochastic convection model**

**Supervisor: Christian Jakob**

**Co-supervisor: Laura Davies**

In climate models thunderstorms must be included as conceptual models often referred to as parametrizations. For those to work there has to be a relationship between what the model equations resolve and the thunderstorms themselves. It has been shown that this relationship has a significant stochastic component. In addressing this several stochastic models of thunderstorms have been proposed, but none of them have been tested against observations. This project will evaluate a stochastic multi-cloud model against observations at Darwin with the goal to derive recommendations how to improve the model.

### **References:**

<http://users.monash.edu.au/~cjacob/>

## Diffusively-driven stratified flows

**Supervisor:** Michael Page

**Co-Supervisor:** To be advised

**Background:** In 1970, two independent studies (by Wunsch and Phillips) of the behaviour of a linear density-stratified fluid in a closed container showed that motion can be generated simply due to the container having a sloping boundary surface, and furthermore that the fluid flows uphill! This remarkable phenomenon is a result of the curvature of the lines of constant density near any sloping surface, in order that a zero normal-flux condition on the density to be satisfied along that boundary.

Since that time a number of studies have since considered the consequences of this type of 'diffusively-driven' flow, including in the deep ocean and with turbulent effects included. More recently, Peacock *et al* (2004) undertook an experimental study of the phenomenon in a closed container and Page & Johnson (2008, 2009) extended the work to consider the broader-scale mass recirculation that is generated.

**Objectives:** The project will review and compare previous analytical, experimental and computational work on this problem for various geometries. The analytical approach introduced in Page & Johnson (2008, 2009) will be used to predict the form of the steady linear flow at the initial stages of the experiments by Peacock *et al* (2004) for various bottom slopes and compare that with computational results from a numerical model of the full governing equations.

**Expectations:**

- The previous work on diffusively-driven flows will be reviewed in relation to the geometries studied and the key non-dimensional parameters.
- An analytical solution will be found for the steady 'outer flow' in a similar configuration to that considered in the laboratory experiments by Peacock *et al* (2004), based on the theory in Page (2011).
- A computational model will be developed using MATLAB for a steady linear driven flow for the same type of container and those results analysed for various angles of inclination of the lower wall.

**Assumed knowledge:**

- MTH3011 Partial differential equations (required); and
- MTH2051/3051 Introduction to computational mathematics (preferred); and
- MTH3360 Fluid dynamics (preferred).

**Reading:**

- Page, M.A.. & Johnson, E.R., On steady linear diffusion-driven flow, *J. Fluid Mech.* **606**, pp433-443, 2008.
- Page, M.A.. & Johnson, E.R., Steady nonlinear diffusion-driven flow, *J. Fluid Mech.* **629**, pp299-309, 2009.
- Page, M.A., Steady diffusion-driven flow in a tilted square container, *Quart. J. Mech Appl. Math.* **64**, pp319-348, 2011.
- Peacock, T., Stocker, R. & Aristoff, J. M., An experimental investigation of the angular dependence of diffusion-driven flow, *Phys. Fluids* **16**, pp3503-3505, 2004.
- Phillips, O.M., On flows induced by diffusion in a stably stratified fluid, *Deep-Sea Research* **17**, pp. 435-443, 1970.
- Wunsch, C., On oceanic boundary mixing, *Deep-Sea Research* **17**, pp293-301, 1970.

**Last reviewed:** 5 September 2011

## **The influence of sea breezes and orography on frontal passages at Cape Grim, Tasmania**

**Supervisor: Steve Siems**

**Co-Supervisor: To be advised**

Background: The Cape Grim Baseline Air Pollution Station (CGBAPS) has been monitoring clean ‘baseline’ air for over 30 years as part of the global effort in understanding changes in greenhouse gas concentrations. The station is also used to monitor ozone-depleting gases and regional air pollution.

The meteorology of Cape Grim (located on the Northwest tip of Tasmania) is dominated by mid-latitude frontal passages. The warm, pre-frontal air mass normally brings in air that has been influenced by the continent, and this is followed by a swift change to post-frontal, ‘baseline’ air that has been travelling over the Southern Ocean and free of anthropogenic influence.

Project Outline: Both the local sea breeze and the mountains of Tasmania confound these ideal frontal passages. Through a series of numerical simulations, these effects will be isolated and identified. The honours student will employ CSIRO’s The Air Pollution Model (TAPM) code. Simulations will then be tested against observations taken at Cape Grim.

### **References:**

Holper, P., “Monitoring the atmosphere at the Cape Grim Baseline Air Pollution Station”, *Clean Air*, 26, pp. 55-62, 1992.

Hurley, P.J., “The Air Pollution Model (TAPM) Version 2: Technical Descriptions and Examples”, CSIRO Atmospheric Research Technical Paper 55, 2002.

Physick, W.L., “Mesoscale modelling of a cold front and its interaction with a diurnally headed land mass”, *J. Atmos. Sci.*, 45, pp. 3169-3187, 1988.

## 5. APPLIED AND COMPUTATIONAL MATHEMATICS

### Staff members - 2012:

Professor Robert Bartnik

*(also member of Analysis and Geometry research group)*

Dr Leo Brewin

*(also member of Astrophysics and General Relativity research group)*

Professor Paul Cally

*(also member of Astrophysics and General Relativity research group)*

Dr Simon Clarke

*(also member of Atmospheric & Geophysical Fluid Dynamics research group)*

Dr Tim Garoni

*(also a member of the Algebra and Discrete Mathematics research group and Statistical Mathematics research Group)*

Dr Jules Kajtar

*(also a member of the Astrophysics and General Relativity research group)*

Professor John Lattanzio

*(also member of Astrophysics and General Relativity research group)*

Dr Rosemary Mardling

*(also member of Astrophysics and General Relativity research group)*

Professor Louis Moresi

*(also member of School of Geosciences)*

Assoc. Professor Michael Page

*(also member of Atmospheric & Geophysical Fluid Dynamics research group )*

Dr Daniel Price

*(also a member of the Astrophysics and General Relativity research group)*

# Projects

## Modelling traffic on road networks using stochastic cellular automata

**Supervisor: Tim Garoni**

**Co-supervisor: Jan de Gier**

Background: In addition to being of significant real-world importance, the study of vehicular traffic has played an increasingly significant role in the development of non-equilibrium statistical mechanics over recent years. The use of cellular automata has been particularly popular within the statistical mechanics community, ever since the introduction of the Nagel-Schreckenberg (NaSch) model. A cellular automaton is a model which is discrete in time, space, and state variables, whose dynamical rules are local. The NaSch model is generally considered to be the minimal model for traffic on freeways. A huge literature dealing with various extensions of the NaSch model has evolved since its first appearance, and our understanding of freeway traffic has benefited greatly as a result. The behaviour of traffic networks, by contrast, is far less well understood. The NetNaSch model, recently developed by our group, provides a simple way to extend the NaSch model to arbitrary urban road networks.

Project Outline: The basic code for the NetNaSch model is already written (in C++), and this project would involve modifying certain subroutines of the code, using the code to run simulations, and then performing statistical analyses of the simulated data. This project would therefore provide training and hands-on experience in scientific computing, mathematical modelling, simulation, and statistical analysis, as well as in relevant aspects of statistical mechanics. Some previous experience in programming would be very useful, but prior C++ experience is not essential. Some specific questions that could be studied are:

- + Compare the efficiency of different traffic signal systems
- + Investigate the interplay between freeway ramp metering and arterial network congestion
- + Study the effect of increased tram priorities at traffic signals

Some more theoretical questions that could also be tackled by simulation are:

- + Study fluctuations of key traffic observables, such as travel time, speed, and queue length, under different signal-control strategies
- + Study the existence of “Macroscopic Fundamental Diagrams”
- + Study parameter sensitivity, and how it relates to fluctuations in key traffic observables
- + Investigate the existence of “scaling”

Our group has been collaborating with VicRoads since 2008, and you would have the opportunity to attend our regular meetings with VicRoads traffic engineers. There would be scope for an investigation that may lead to new results.

## References:

- de Gier, J, Garoni, T and Rojas, O “Traffic flow on realistic road networks with adaptive traffic lights”, *J. Stat. Mech.* P04008 (2011).
- Schadschneider, A, Chowdhury, D and Nishinari, K “Stochastic transport in complex systems: From molecules to vehicles”, Elsevier (2010).

## Simulations of swimming fish

**Supervisor: Jules Kajtar**

**Co-Supervisor: Joe Monaghan**

**Background:** We have developed a code to numerically model the motion of fish-like swimmers moving through a fluid [1,2]. The fluid is modelled using Smoothed Particle Hydrodynamics, and the swimmer is a series of linked, rigid bodies.

**Objectives:** A number of interesting questions have arisen from these studies. Students may choose one of the following topics to form the basis of a research project:

1. *Fish Shoaling:* How does the efficiency of two or more swimmers change when they swim in different configurations, e.g. in line or side-by-side?
2. *Optimal Shape:* How do the shapes, sizes, and the mass distribution affect the motion of a swimmer?
3. *Leaping and Diving:* Dolphins often swim long distances by periodically leaping out of the water. Are they being playful, or is there a gain in swimming efficiency?
4. *Predator versus Prey:* How can the gait specifications for forward and turning motions be combined in order to track an object moving in an unpredictable fashion? Conversely, what is the best way to plan an escape path?
5. *The Effect of Density Stratification:* Fish normally swim in a body of water that is saltier at the bottom. Motion near the interface between fresh and salt water produces waves. Does this mean fish swim less efficiently?

**Expectations:** A generic code exists, to which the student will add new routines. Analysis of the results.

**Assumed Knowledge:**

- MTH3011 — Partial Differential Equations;
- MTH3/2051 - Introduction to Computational Mathematics (or equivalent programming experience).

**References :**

- [1] Monaghan, J.J., Smoothed particle hydrodynamics, Rep. Progress Phys. 68 (2005) 1703-1759.  
[2] Kajtar, J.B., Monaghan, J.J., SPH simulations of swimming linked bodies, J. Comput. Phys. 227 (2008) 8568-8587.

## Extending polytropic models

**Supervisor:** John Lattanzio

**Co-Supervisor:** Paul Cally

**Background:** Simple polytropic models have many uses in stellar astrophysics. The simple approach used in 3rd year can be extended by adding an equation for the luminosity as well. In this case one can include the H burning reactions and calculate a luminosity for the model.

**Project Outline:** The idea is to try to construct polytropic models that represent the main phases of stellar evolution around the HR diagram. How can we best simulate a main sequence? Can we simply use the pp and cno cycle reactions? Will it be necessary to make some modifications? eg are all ms stars equally well approximated by polytropes with  $n=3$ ? Maybe we will need to vary  $n$ ? Can we simulate a red-giant somehow? Further, what is the best way to solve the Lane-Emden equation? We will investigate using a Runge-Kutta-Fehlberg scheme with a maximum specified error at each step. We can also write the Lane-Emden equation as a DE for  $x_i$  as a function of theta, so that the boundaries are now well known!

### References:

See third year ASP3012 notes and John Lattanzio.

## Exploring the dynamics and simulation of rotating linked rigid bodies

**Supervisor:** Joe Monaghan

**Co-Supervisor:** Jules Kajtár

### Level: Honours

**Background:** The dynamics of a rotating rigid body is fascinating. Think of spinning tops. When there are several bodies linked together the motion it is even more interesting and its description more challenging. Apart from ordinary differential equations it can involve rotation matrices and objects called quaternions. It is central to our current ARC project on swimming linked bodies though it can be considered independently.

**Objectives:** To write the equations of linked rigid bodies in an economical way, and in such a form that accurate simulations can be made efficiently, and to use the simulations to study their dynamics.

### Expectations:

- An understanding of the dynamics of a single rotating arbitrary rigid body.
- The ability to derive the equations of motion for several linked bodies.
- To write a program to simulate rigid linked bodies with various couplings between them.
- To analyze the dynamics of the linked bodies.

### Assumed Knowledge:

MTH2032 | Differential equations

A knowledge of mechanics.

Introduction to Computational Mathematics (or equivalent programming experience).

### Reading:

Landau and Lifshitz, Mechanics, Vol 1. "Course of Theoretical Physics".

## The bathtub vortex

**Supervisor:** Michael Page

**Co-Supervisor:** To be advised

**Background:** It is a common myth that the direction of rotation of bath water as it drains down a plughole is determined by the rotation of the Earth – in particular the hemisphere in which it is performed. Careful experiments have been performed to confirm that the rotation of the Earth has only a miniscule effect on the motion (Shapiro, 1962; Trefethen *et al*, 1965) and it can also be demonstrated theoretically using simple scaling arguments. More recently, Tyvand & Haugen (2005) published a computational study of the problem and also demonstrated that the influence of the rotation of the Earth is negligible. But why does the water often flow down the plughole in a consistent direction, and why is that direction opposite in the Australia to what is commonly observed in the United Kingdom, for example?

**Objectives:** The project will review previous analytical, experimental and computational work on this problem for simplified geometries. Among other things, this will help identify the principal determinants of the direction of motion of the fluid at the outlet and assist in identifying the key physical principles based on a mathematical analysis of the equations of motion.

**Expectations:**

- To undertake a thorough review of existing primary literature sources on this problem.
- To identify the dominant forces that affect the fluid motion, derive the corresponding governing equations and assess the relative size of the relevant nondimensional parameters.
- To simplify and solve the equations in appropriate cases, consider whether simplified models can adequately represent the phenomenon and perhaps undertake simple computational experiments.

**Assumed knowledge:**

- MTH3011 Partial differential equations; and
- MTH2051/3051 Introduction to computational mathematics; and
- MTH3360 Fluid dynamics.

**Reading:**

- Andersen A. *et al*, Anatomy of a bathtub vortex, *Phys. Review Letters* **91**, 104501, 2003.
- Shapiro, A.H., Bath-tub vortex, *Nature* **196**, pp1080-1081, 1962.
- Trefethen, L.M. *et al*, The bath-tub vortex in the southern hemisphere, *Nature* **207**, pp1084-1085, 1965.
- Tyvand, P.A. and Haugen, K.B., An impulsive bathtub vortex, *Phys. Fluids* **17**, 062105, 2005.
- Yukimoto, S. *et al*, Structure of a bathtub vortex: importance of the bottom boundary layer, *Theor. Comput. Fluid Dyn.* **24**, pp323-327, 2010.

**Last reviewed:** 5 September 2011

## Resolving corner singularities in viscous flow

**Supervisor:** Michael Page

**Co-Supervisor:** To be advised

**Background:** The solution of PDEs in domains with sharp corners can often involve singularities at those corners, where either the solution or some of its higher derivatives may become infinite as the corner is approached. A simple example of this is the solution for very viscous flow past a thin flat plate, for which Carrier and Lin (1948) demonstrated the velocity is proportional to  $r^{1/2}$  as  $r \rightarrow 0$  near the end of the plate. Similar types of behaviour can also occur near angular corners (Moffatt, 1966).

Numerical methods for solving PDEs typically assume that the solution is ‘well-behaved’ with all derivatives finite at every point in the domain and on the boundary, so they need to be modified when singularities are known to be present. A recent paper by Shi et al (2004) describes one way of doing this in the case of one particular viscous-flow problem, where the solution satisfies the ‘bi-harmonic equation’  $\nabla^4 \psi = 0$  and the domain involves an infinitely-long flat plate. The aim of this project is to compare their method with some other possible approaches that also take into account our knowledge about the nature of the singularity at the leading edge of the plate.

**Objectives:** The project will commence with a review of some of the existing primary literature sources on both the nature of singularities of viscous flows at corners and the treatment of no-slip boundary conditions in numerical models of those problems. The approach used by Shi et al (2004) will be studied in detail for the infinite plate problems and their results reproduced for some of the cases which they have considered. Their results will also be compared with some other simpler approaches. If time permits, the results will be extended to flow near corners of varying angles, including those at right-angles.

**Expectations:**

- A review will be undertaken of some of the existing primary literature sources on viscous-flow singularities at corners in order to understand the properties which are required to be resolved in the numerical solutions.
- The method used by Shi et al (2004) will be examined in detail and implemented for some of the test problems undertaken in their study. This method will also be compared with some other possible approaches that also take into account the nature of the singularity.
- The outcome of the project will be a working computational model for the solution of viscous flow past a flat plate with the singularities at the leading edge resolved accurately. If possible, the method will be extended to flow near corners of varying angles, including those at right-angles.

**Assumed knowledge:**

- MTH3011 Partial differential equations; and
- MTH3020 Complex analysis and integral transforms; and
- MTH2051/3051 Introduction to computational mathematics.

**Reading:**

- Carrier, G.F. and Lin, C.C., On the nature of the boundary layer near the leading edge of a flat plate, *Quart. Appl. Math.* **6**, pp63-38, 1948.
- Moffatt, H.K., Viscous and resistive eddies near a sharp corner, *J. Fluid Mech.* **18**, pp1-18, 1966.
- Shi, J.-M *et al*, A combined analytical-numerical method for treating corner singularities in viscous flow predictions, *Int. J. Numerical Meth. Fluids* **45**, pp659-688, 2004.

**Last reviewed:** 5 September 2011

## On the roll-up of a ‘vortex sheet’

**Supervisor:** Michael Page

**Co-Supervisor:** To be advised

**Background:** The position of a thin interface between two fluids that are moving at different speeds is known to be unstable in an inviscid fluid. This is known as Kelvin-Helmholtz instability, and typically it leads to counter-rotating vortices on the interface. Such vortices are often observed between atmospheric layers, and also in the wake of moving vehicles.

In this project the Kelvin-Helmholtz instability is considered for an idealised problem of a thin straight interface between two infinite fluids that have different constant velocities parallel to the interface. For a two-dimensional inviscid fluid this problem can be posed in the complex plane and determined computationally using complex-valued functions (see for example Krasny 1986 and Baker & Pham 2006). It can also be shown that the subsequent roll-up of the vortex develops as the singularity of one of the functions approaches the real axis, where the interface is initially located.

**Objectives:** The project will review and compare some of the existing primary literature sources on vortex roll-up and the desingularisation of the equations, including both the approach of Krasny (1986) and the introduction of additional factors, for example including viscosity. Krasny’s results will be reproduced and then extended to other initial conditions. Key features in the roll-up will be identified beyond the critical time at which it ‘overturns’ and begins to form a spiral shape.

**Expectations:**

- A review will be undertaken of some of the existing primary literature sources on this problem in order to identify the current state of knowledge.
- The equations that determine the motion of the interface will be derived and the corresponding complex-plane formulation justified.
- MATLAB (for example) will be used to recalculate and extend the results of the periodic problem in Krasny (1986), where a small parameter  $\delta$  was introduced in order to ‘desingularise’ the interface.
- Krasny’s approach will be extended to the simpler problem of the roll-up due to an isolated disturbance, for example using recent work by Baker & Pham (2006).

**Assumed knowledge:**

- MTH3011 Partial differential equations (required); and
- MTH3020 Complex analysis and integral transforms (required); and
- MTH3360 Fluid dynamics (preferred).

**Reading:**

- Baker, G.R. and Pham L.D., A comparison of blob methods for vortex sheet roll-up, *J. Fluid Mech.* **547**, pp297-316, 2006.
- Krasny, R., Desingularisation of periodic vortex sheet roll-up, *J. Comp. Phys.* **65**, pp292-313, 1986.

**Last reviewed:** 5 September 2011

## The Immersed Interface Method for solving elliptic PDEs

**Supervisor:** Michael Page

**Co-Supervisor:** To be advised

**Background:** Most of the simple approaches to solving elliptic partial differential equations numerically using finite-difference techniques rely upon the solution being smooth and continuous everywhere in the domain. In addition, the domain must have a simple, regular geometry – such as a rectangle or a circle. Over the last decade or so, a couple of approaches have been developed which allow discontinuities of the solution and/or consider domains with an irregular shape. In the latter case, these methods can be a simpler alternative to using finite-element methods.

The two most common approaches used for these problems are the Immersed Boundary Method, which was first developed by Charles Peskin in the 1970s, and the Immersed Interface Method. Some aspects of these techniques are similar, but there are important differences. This project examines the basis of the Immersed Interface Method and uses it to examine the accuracy of the approach for some simple test problems with exact solutions.

**Objectives:** The project will review and compare some of the existing primary literature sources on both the Immersed Interface Method and the Immersed Boundary Method. Some trials of the Immersed Interface Method will be undertaken, initially based on the test problems in Li and Ito (2006) but then extended to a broader range of configurations, with the intention of identifying the advantages and limitations of the method.

**Expectations:**

- Primary literature sources on the Immersed Interface Method, including LeVeque and Li (1994), and the Immersed Boundary Method will be reviewed, and the two approaches compared in detail.
- The material covered in the first three chapters of Li and Ito (2006) will be examined in detail and all numerical results checked using MATLAB. Some additional simple one and two-dimensional test problems involving elliptic PDEs will also be solved using a similar approach in order to test the capabilities of the method.
- The approach will be applied to some simple ‘embedded’ boundary-value problems in two-dimensional domains with internal boundaries.

**Assumed knowledge:**

- MTH3011 Partial differential equations (required); and
- MTH2051/3051 Introduction to computational mathematics (preferred).

**Reading:**

- LeVeque, R.J. and Li, Z., The immersed interface method for elliptic equations with discontinuous coefficients and singular sources, *SIAM J. Numer. Anal.* **31**, pp1019-1044, 1994.
- Li, Z. and Ito, K., The Immersed Interface Method, *SIAM*, 2006.

**Last reviewed:** 5 September 2011

## **Sub-resolution models for supersonic turbulence**

**Supervisor: Daniel Price**

**Co-Supervisor: Joe Monaghan**

Turbulence is hugely important in many physical problems, including many in astrophysics. However, the turbulence in star-forming molecular clouds is quite different to laboratory turbulence because the motions far exceed the speed of sound. Thus, whilst a great many "sub-grid" models have been developed for simulating turbulence in the laboratory on a computer, it is not clear whether any of these can be applied in the supersonic regime.

In this project we will attempt to extend a sub-resolution turbulence model developed for the Smoothed Particle Hydrodynamics method [1,2] to deal with supersonic flows. We will consider some simple problems in one dimension whereby a shock interacts with a "turbulent" medium in order to demonstrate whether or not the method can be applied in this regime and whether or not it adequately captures the sub-resolution physics that occurs in turbulent flows. Extensions to the project could include the application to more realistic flows in two or three dimensions.

The project would suit a student in either astrophysics or applied maths who enjoys computing. Successful completion of the project has the potential to lead to a full research publication, so would be a good choice for a student considering further PhD research.

### **References**

Monaghan, J. J., *Journal of Turbulence*, Volume 5, Issue 1, pp. 012 (2004).

Monaghan, J. J., *Monthly Notice of the Royal Astronomical Society*, Volume 335, Issue 3, pp. 843-852 (2002)

## **Projects in Collaboration with the Computational Fluid Dynamics (CFD) Group at CSIRO Mathematical and Information Sciences (CMIS)**

The following is a list of proposed honours projects being offered by the School of Mathematical Sciences in collaboration with the Computational Fluid Dynamics (CFD) Group at CSIRO, Mathematical and Information Sciences (CMIS).

Research areas that are of interest to the CFD group at CSIRO, Mathematical and Information Sciences (CMIS) are listed below. Defined honours projects in these areas of research can be undertaken through our School of Mathematical Sciences in collaboration with the CFD group at CSIRO.

Each of the project suggestions below would have a mix of the following components: - literature review, small amounts of theoretical development, small amounts of code writing to capture new capabilities, running of several simulations, comparisons with test or experimental data and the preparation of a written report/thesis.

All projects would use the CMIS DEM and SPH codes as a basis.

All enquiries regarding these projects should be through Gerald Pereira [per233@csiro.au](mailto:per233@csiro.au) after first seeking approval with the Honours Coordinator [Todd.Oliynyk@monash.edu](mailto:Todd.Oliynyk@monash.edu).

## **Effect of particle shape on the shear bands characteristics in granular materials**

**CSIRO supervisors:           Dr Vincent Lemiale, Dr Gary Delaney & Dr Gerald Pereira**

**Monash collaborators:       Louis Moresi (School of Maths), Sandy Cruden (School of Geosciences)**

Although granular materials are ubiquitous in nature, a complete understanding of their mechanical behaviour has not been achieved yet. An important aspect of this behaviour is the observed tendency for localization in granular materials. Indeed macroscopic shear bands are commonly observed as the mechanical response of the material to an applied load. A vast amount of research has been dedicated to the understanding of shear band formation in granular materials, particularly in the last few decades. In addition to experimental studies, numerical work has been undertaken using discrete element methods (DEM) which aim at modelling the material at the scale of the grain. The advantage of conducting numerical simulations over experiments lies in the large quantity of information that can be extracted from any given simulation. Such detailed temporal and spatial data is not always available in experiments.

The abundant literature on the issue of shear banding in granular materials has highlighted the importance of the microscopic characteristics of the granular assembly on the macroscopic features of the bands. For example, it has been shown that the band width is directly influenced by the grain size. Other factors have been shown to play an important role in shear banding, most notably the rotation of the grains within the band. This latter point is essential because the vast majority of numerical modelling using DEM assumed a spherical shape for the grains. Intuitively, this shape will tend to favour grain rotations and possibly exaggerate its effect compared to more realistic grain shapes. In this study, we will investigate the effect of granular shape on the characteristic of shear bands by means of numerical modelling using our in-house DEM code. We will focus on a non cohesive material. A numerical analogue of a simple shear experiment will be used in the simulations. Some experiments have been or will be conducted at Monash University so a direct comparison with experiments might be possible.

## **Mathematical modelling of complex fluids**

**CSIRO supervisor:   Dr Gerald Pereira**

Complex fluids encompass a range of fluids which do not show Newtonian behaviour (a constant viscosity fluid). Examples of complex fluids in everyday life include honey, paints, oils to name but a few. In real world applications fluids will flow with free deformations of their surfaces. For example, when pouring a liquid on to a solid surface, the liquid will form a (roughly) cylindrical jet, before breaking up into a myriad of droplets of various shapes and sizes. Mathematical modelling of this behaviour is difficult (for example by using the Navier-Stokes equation) not least because of the multiple surfaces and interfaces that are involved. In this project we will look at modelling an exotic complex fluid – a liquid crystalline material. This material naturally forms layered phases (a so called smectic) and our focus is to model the phase behaviour of this smectic material when poured on a solid surface, using a meshless computer method. By meshless we mean we will solve the partial differential equations, which govern the system, without a specified lattice domain. This can be particularly advantageous for freely forming liquid crystals.

## **Effect of Particle Shape on Alluvial and Gravity Driven Granular Bedforms**

**CSIRO supervisors: Dr James Hilton & Dr Gary Delaney**

Wind and water flowing over granular beds act, along with gravity, to produce a diverse range of bedforms including dunes, waves and ripples. This project will investigate the influence of the shape of the underlying particles on the overall form of the dynamic bed, and look for correlations between micro-scale and macro-scale structures within the bed. The project will involve numerical simulations and analysis, as well as the development of theoretical models. The first part of the project will be the set-up and modelling of granular bed flow, using computational simulations of large numbers of shaped particles interacting with a flowing fluid. The second part will be the development of spectral analysis techniques to categorise and investigate the resulting bedforms. The final part of the project will be the investigation of theoretical models linking features of the bedform to properties of the particles at the granular level. This will be a challenging and interesting project ideally suited to a student with a strong mathematical background, with an interest in physical systems.

## **Modelling void formation and jet collapse using SPH**

**CSIRO supervisor: Dr Mahesh Prakash**

Some fluid structure interaction problems involve creation of a void as the structure starts interacting with the fluid and collapse of the fluid jet after the penetration of the structure into the fluid. There is intense deformation of the fluid free surface during this process. In the experiment, a long smooth rod with a disk shaped end is dragged at varying speeds into standing water creating a void in the fluid. The void then closes resulting in a jet that travels above the fluid free surface and finally collapses. This project involves performing 2D and 3D SPH simulations replicating the simulations at various Froude number values and comparing the results to determine the ability of SPH to simulate such phenomenon starting from the void formation to the jet formation and finally collapse of the jet. A boundary integral approach tried earlier was not able to simulate the latter part of the fluid structure interaction phenomenon.

## **Projects offered jointly with School of GeoSciences and School of Mathematical Sciences**

### **The dynamics of continental crust in early Earth, on Venus and Mars - *Joint with Geosciences***

**Supervisor: Louis Moresi**

**Co-Supervisor: A. Lenardic (Rice University, Texas)**

**Field of Study: Geodynamics, Planetary Science, Computer Modelling.**

**Project Outline:** This project involves numerical modelling of thermochemical convection which simulates continental crust coupled to the convecting mantle. There are a number of regimes of behaviour which can result from the competition between chemical and thermal buoyancy as the Earth cooled through time it is likely that the high Rayleigh number regimes were more important than today. This may give some insight into the formation of proto-continent and the nature of tectonics in the very early Earth. This work will also have application to the tectonics of Mars and Venus which will give the student an opportunity to work as part of an international team looking at the dynamics of these planets.

### **References:**

See Louis Moresi, [Louis.Moresi@monash.edu](mailto:Louis.Moresi@monash.edu)

## **Modelling the dynamics of subduction zones**

**Supervisor:** Louis Moresi

**Co-supervisors:** Dave Stegman and Pete Betts

**Field of study:** Geodynamics / global tectonics

Support offered: Supercomputer training course

Collaborating organisations: Victorian Partnership for Advanced Computing

Project Outline: This project brings together computer modeling and large scale geophysical datasets such as seismic tomography, oceanic bathymetry and geoid, regional seismicity to model how slabs evolve during the subduction process. No programming is required but there is the opportunity to learn about how computational geodynamics codes work. The project can be taken in a number of different directions according to the interest and experience of the student. For example: comparing geological observations in convergent settings with model results; compiling remote sensed observations and tectonic reconstructions for a specific region to build your own model; studying the stress conditions in the slab for a range of parameter space.

### **References:**

See Louis Moresi, [Louis.Moresi@monash.edu](mailto:Louis.Moresi@monash.edu)

## **Analogue and numerical models of crustal deformation models**

**Supervisor:** Louis Moresi

**Co-Supervisor:** To be Advised

**Field of study:** Geodynamics / structural geology

Support offered: Laboratory materials

Collaborating organisations: Victorian Partnership for Advanced Computing

Project Outline: This project is aimed at better understand the dynamics of extension and/or compression of the continental lithosphere. It will involve Earth-like experiments in the Epsilon laboratory combined with numerical models using the Underworld code. No programming is required but there is the opportunity to learn about how computational geodynamics codes work.

The project can be tailored to the student's individual interest (e.g. if you have a preferred field area which you would like to model). In general, however, the approach is similar: first we expect to show that the parametrization used in the numerical experiments is relevant when compared with the laboratory experiments. Second we wish to find the different regimes of behaviour for the extension and/or compression of the continental lithosphere in terms of the typical scales that will characterize our models. Comparison with regions responding to compression and/or stretching can also be investigated.

### **References:**

See Louis Moresi, [Louis.Moresi@monash.edu](mailto:Louis.Moresi@monash.edu)

## 6. STATISTICAL MATHEMATICS RESEARCH GROUP

### Staff members - 2012:

Dr Davaatseren Baatar

Dr Tim Garoni (*also a member of the Algebra and Discrete Mathematics Research Group and Statistical Mathematics Research Group*)

Dr Kais Hamza

Dr Jonathan Keith

Professor Fima Klebaner

Dr Greg Markowsky

Dr Boris Miller

Professor Kate Smith-Miles (Operations Research)

Dr Tianhai Tian

Dr Daniel Tokarev

## Projects

### Fixed Interval Scheduling

**Supervisor: Davaa Baatar**

**Co-Supervisor: Kate Smith-Miles**

Background: The main defining characteristic of the problem is scheduling a number of jobs with fixed processing interval on a given number of certain type of machines. Each machine can handle only certain(predefined) set of jobs. Multi-tasking and preemption are not allowed.

Fixed interval scheduling problem(FISP) is shown to be NP-hard. Different variations and special cases of the FISP are widely studied in last two decades due to its wide range of applications in real-world problems such as bus driver scheduling, rostering, satellite data transmitting, fleet planning, gene identification, examining computer memory structure etc.

FISP is closely related in well known problems in graph theory and in some special cases it can be presented as k-coloring of interval graph, circular arc graph and periodic interval graph.

Project outline: Main contribution of this project will be developing general formulation of the FISP, classifying the special cases, implementing and testing the existing algorithms, and developing new algorithms for some special cases of the problem.

### References:

Kovalyov, M.Y, Ng, C.T. and Cheng, T.C.Edwin. "Fixed interval scheduling: Models, applications, computational complexity and algorithms", European Journal of Operational Research, 178:331-342, 2007.

## **Markov-chain Monte Carlo methods in Statistical Mechanics**

**Supervisor:**           **Tim Garoni**

**Co-supervisor:**       **Daniel Tokarev**

Background: Statistical mechanics began life as a branch of mathematical physics, but is now a central paradigm for studying all manner of complex systems, across fields as diverse as physics, chemistry, biology, economics and sociology. An important branch of statistical mechanics concerns discrete models, in which one studies random structures defined on graphs. These studies have significant overlap not only with probability theory, but also combinatorics and computer science. Since models in statistical mechanics are often mathematically intractable, Markov-chain Monte Carlo (MCMC) methods have become an indispensable computational tool in this field.

However, not all Markov chains are created equal, and while two Markov chains may have the same stationary distribution (and therefore approximate the same statistical mechanical model), their rates of convergence to stationarity (and therefore their practical efficiency) can be very different. While the classical theory of Markov chains considers the late-time asymptotics of fixed chains, the relevant asymptotics in statistical mechanics concerns the growth of “mixing times” as the size of the state space becomes large. Quantifying the size of such mixing times, and designing new Markov chains with reduced mixing times, are the central tasks in this field.

Project Outline: The project would start with a review of the required basic background in discrete statistical mechanics, and Markov-chain Monte Carlo; the references listed below being a good place to start. From there, a range of possible projects is available, studying specific classes of Monte Carlo methods for specific classes of statistical-mechanical models. Depending on your interests, the project will involve a combination of both theoretical studies and computer experiments; the focus could range from being largely computational, to entirely theoretical. There would be scope for an investigation that may lead to new results.

### **References:**

Levin, A, Peres, Y, and Wilmer, E “Markov Chains and Mixing Times”, available online free (legally) at <http://pages.uoregon.edu/dlevin/MARKOV/markovmixing.pdf>

Recorded lectures at <http://www.msri.org/web/msri/scientific/show/-/event/Wm406> and <http://www.msri.org/web/msri/scientific/show/-/event/Wm318>

Grimmett, G “Probability on Graphs”, Cambridge University Press (2010), available online free (legally) at <http://www.statslab.cam.ac.uk/~grg/books/pgs.html>.

## Optimising in-stream nutrient uptake studies

**Supervisor: Kais Hamza**

**Co-supervisor: Dr Mike Grace (Water Studies Centre, School of Chemistry)**

**Project Outline:** This project will investigate the optimization of the nutrient spiralling technique that is commonly used in stream ecology to assess the ability of the stream environment to take up nutrients and therefore minimise water quality problems, including toxic algal blooms, in the receiving waters (lakes, estuaries). The technique typically involves adding a nutrient (nitrogen or phosphorus) along with an inert tracer (bromide) to the stream and following its disappearance between two sampling stations. Recent work in the Water Studies Centre has shown that many of the conclusions drawn in the literature about the behaviour of streams is severely compromised by a lack of explicit consideration of uncertainty (Hanafi et al. 2007).

Nutrient uptake is described by two equations:

The uptake length,  $S_w$  (m), the average distance traveled by a nutrient atom before it is taken up into the sediment (by plants or abiotic processes) is defined as:

$$S_w = \frac{x}{\ln\left(\frac{C_{Nu-Up}}{C_{Nu-Do}}\right) - \ln\left(\frac{C_{Br-Up}}{C_{Br-Do}}\right)} \quad (1)$$

- (1) Where  $x$  is the study reach length (m);  $C_{Nut-Up}$  and  $C_{Nut-Do}$  are the mean nutrient concentrations at the upstream and downstream stations respectively ( $\mu\text{g/L}$ ); and  $C_{inert-Up}$  and  $C_{inert-Do}$  are the inert tracer concentrations at the upstream and downstream stations respectively ( $\mu\text{g/L}$ ).

Optimising in-stream nutrient uptake studies continued over the page \...

Optimising in-stream nutrient uptake studies continued from previous page\...

- (2) To remove the effect of stream flow and width on uptake, the uptake velocity,  $V_f$  (m/s) is calculated from the

following relationship: 
$$V_f = \frac{Q}{wS_w}$$

where  $Q$  is the measured stream discharge (in units of  $\text{m}^3/\text{s}$ ) and  $w$  is mean stream width (m).

Uncertainties can then be calculated by replacing single values in equations 1 and 2 with distributions, represented by means and standard deviations. Monte Carlo methods are then applied to characterize the distributions for  $S_w$  and  $k_f$ .

This project will extend this initial uncertainty analysis to consider key questions including:

What is the optimum number of sampling stations required to measure the spiralling metrics with a given precision? (e.g. specified 95% confidence intervals). A penalty function will need to be applied for each additional sampling station.

How many samples need to be taken to obtain this required level of precision?

What is the best spacing for the sampling points - equidistant?

Consideration of real heterogeneity in behaviour (as the real streams under consideration are not uniform channels)

## References:

Hanafi, S., M. Grace, J.A. Webb and B. Hart (2007). "Ecosystems", 10, 477-487.

## **Financial mathematics**

**Supervisor: Kais Hamza**

**Co-Supervisor: Fima Klebaner**

Project Outline: There are many topics to choose from. Topics vary greatly in the degree of theoretical and practical work involved. Students can choose a topic with any mixture of the theory of stochastic calculus (used in the modelling of financial markets), and the practice of statistical analysis (applied to real data from the Australian market).

### **References:**

Hull, J., "Options, futures and other derivative securities", Prentice Hall, 1989.

Klebaner, F.C., "Introduction to stochastic calculus with applications, Imperial College Press", 1998.

## **Characterisations and probability distributions**

**Supervisor: Kais Hamza**

**Co-Supervisor: To be advised**

Project Outline: The normal and exponential distributions play essential roles in probability and statistics. This project aims at reviewing the main characterisation results and how these are used in various areas of probability and statistics.

### **References:**

Galambos, János & Kotz, Samuel, "Characterizations of probability distributions: a unified approach with an emphasis on exponential and related models", Springer-Verlag, 1978.

Patel, J.K. & Read, C.B., "Handbook of the normal distribution", Dekker, 1996.

## **Probability, a measure theory approach**

**Supervisor: Kais Hamza**

**Co-Supervisor: To be advised**

Project Outline: Probability (or stochastic) models are very widely used. A good understanding of the basic probability theory is an absolute necessity. This topic will cover the basic measure theory (measurability, integrability ...) as well as concepts specific to the area of probability theory (independence, conditioning, distributions...).

### **References:**

See Kais Hamza, [Kais.Hamza@monash.edu](mailto:Kais.Hamza@monash.edu)

## **Learning about genes using Bayesian classification**

**Supervisor: Jonathan Keith**

**Co-Supervisor: To be advised**

Project Outline: The human and other genomes contain many regions for which there is evidence of a functional role but which do not code for proteins. Although some of these non-protein-coding genes have been characterised, many more remain poorly understood. One technique for learning about such genes is to organize them into classes of similar or related sequences, so that knowledge about one member can illuminate other members of that class. This project will explore methods of classifying such genes, and ways of using such classifications to expand knowledge.

### **References:**

Contact Dr Jonathan Keith, [Jonathan.Keith@monash.edu](mailto:Jonathan.Keith@monash.edu)

## **Detecting DNA motifs that drift**

**Supervisor: Jonathan Keith**

**Co-Supervisor: To be advised**

Project Outline: DNA motifs are short patterns in DNA sequences that play a functionally important role. Recent studies have found that the ability to detect such motifs is enhanced using new algorithms that combine data from multiple species. However, these algorithms require motifs to remain similar over evolutionary time, and do not allow for motifs that change with time (drift). This project will trial several new statistical methods for detecting motifs that drift.

### **References:**

Contact Dr Jonathan Keith, [Jonathan.Keith@monash.edu](mailto:Jonathan.Keith@monash.edu)

## **Modelling DNA evolution under biological constraint**

**Supervisor: Jonathan Keith**

**Co-Supervisor: To be advised**

Project Outline: Many functional parts of the genome, including non-protein-coding RNAs, are not free to evolve neutrally, but are constrained by the need to maintain their function. This project will involve developing new statistical models for evolution of biological sequences under such constraints. One use of such models is to identify parts of genomes that are evolving under constraint, and which are therefore likely to be functional. Another use is to enhance phylogenetic algorithms (that is, algorithms that estimate the evolutionary tree of a set of sequences).

### **References:**

Contact Dr Jonathan Keith, [Jonathan.Keith@monash.edu](mailto:Jonathan.Keith@monash.edu)

## **Associations between phenotypes and biological networks**

**Supervisor: Jonathan Keith**

**Co-Supervisor: To be advised**

Project Outline: A phenotype is an observable biological characteristic of an individual organism. Although some phenotypes are determined by a single genetic locus or gene, many are the result of contributions from multiple loci. It is likely that many of the loci that contribute to a particular phenotype will belong to the same biological network or system. This project aims to exploit this supposition by identifying statistically significant associations between phenotypes and genetic variations within the member genes of a biological network.

### **References:**

Contact Dr Jonathan Keith, [Jonathan.Keith@monash.edu](mailto:Jonathan.Keith@monash.edu)

## **Identifying biological networks via graph-based clustering algorithms**

**Supervisor: Jonathan Keith**

**Co-Supervisor: To be advised**

The biological literature is vast and contains much information about interactions between individual genes. In principle, biological networks could be inferred by combining information about individual gene interactions that is scattered throughout the literature. In this project, interactions reported in the literature will be represented by a graph in which nodes correspond to genes and edges correspond to interactions. We will explore the use of graph-based clustering algorithms and network motif detection algorithms to extract networks from this graph.

### **References:**

Contact Dr Jonathan Keith, [Jonathan.Keith@monash.edu](mailto:Jonathan.Keith@monash.edu)

## **Modelling the Brisbane fire ant invasion**

**Supervisor: Jonathan Keith**

**Co-Supervisor: To be advised**

Fire ants were accidentally introduced in the Brisbane area over a decade ago, and intensive efforts to eradicate them are still ongoing. This project will further develop existing Bayesian models of the spread of these ants and will contribute to eradication efforts.

### **References:**

Contact Dr Jonathan Keith, [Jonathan.Keith@monash.edu](mailto:Jonathan.Keith@monash.edu)

## **Bayesian network inference in systems biology**

**Supervisor: Jonathan Keith**

**Co-Supervisor: To be advised**

The growth of cells is tightly regulated by an intricate network of biological molecules. Experimental techniques allow us to measure the actions of these molecules, but do not tell us exactly how they work together to regulate growth - this information needs to be inferred from the experimental data.

This project will develop Bayesian latent variable models to infer the structure of metabolic networks, using experimental data from yeast. The project will focus on integrating and modelling diverse data, recreating known structures, and identifying potentially uncharacterised network structures

### **References:**

Contact Dr Jonathan Keith, [Jonathan.Keith@monash.edu](mailto:Jonathan.Keith@monash.edu)

## **Stochastic simulation and evaluation of financial market models**

**Supervisor: Fima Klebaner**

**Co-supervisor: To be Advised**

Background: Mathematical finance is dominated by stochastic models [1]. Spurred by the enormous success of financial derivatives, there is a strong need to develop sophisticated computational techniques for the evaluation of financial market models. Numerical methods are especially important when market models are getting more and more complex. Advances in numerical methods for solving stochastic differential equations and algorithms for evaluating financial market models give rise to opportunities in the application of computational finance.

Project outline: The aim of the honours projects is for the student to gain experience with advanced computational techniques for simulating financial models together with computing programming skills in C++ and MATLAB. There are two possible projects under this topic

(1) You may implement numerical methods for evaluating option values such as the American option [3, 4]. You will study how to evaluate financial market models in a computational environment.

(2) You may discover how to use numerical methods to solve stochastic differential equations, and to discuss the relative performance of different methods in terms of accuracy and efficiency [2, 5]. At the same time, you learn about the theoretical background to the methods.

### **References**

Klebaner, F.C., "Introduction to stochastic calculus with applications", Imperial College Press, 2005.

Kloeden, P.E., Platen, E. and Schurz, H., "Numerical solution of SDE through computer experiments", Springer-Verlag, 1994.

Seydel, R., "Tools for computational finance", Springer, 2002.

Tian, T and K.Burrage, "Accuracy issues of Monte-Carlo methods for valuing American options", ANZIAM J. 44(E), C739-758, 2003.

Tian, T. and Burrage, K., "Implicit Taylor methods for stiff stochastic differential equations", Applied Numer. Maths., 38, 167-185, 2001.

## **Stochastic simulation of biochemical reaction systems**

**Supervisor: Fima Klebaner**

**Co-supervisor: To be Advised**

Background: There is a growing body of evidence which suggests the dynamics of biological systems in the cell, especially genetic regulation, is stochastic. One of the major reasons is the small molecular numbers of proteins in the cell such as transcriptional factors and message RNA. Biochemical reaction systems are typically studied using the stochastic simulation algorithm (SSA). Recent progress in computational biology has proposed mathematical models for large-scale complex biological systems. There is a strong need to develop efficient and effective numerical methods for simulating the dynamics of stochastic biological systems.

Project outline: This project commence with the implementation of the SSA for simulating genetic regulatory networks. Then you will implement more efficient methods such as the tau-leap methods and multi-scale simulation methods. There are two possible projects under this topic.

(1) You may study the dynamic property of a specific biological network in genetic regulation or cell signalling transduction by using stochastic simulation methods. An interesting question of this project is the function of noise in maintaining bistability property of gene networks.

(2) You may study effective simulation techniques, including the implementation of stochastic simulation on high performance computers, to simulating biochemical reaction systems.

### **References**

- Klebaner, F.C., "Introduction to stochastic calculus with applications", Imperial College Press, 2005.  
Gillespie D.T., "Stochastic simulation of chemical kinetics", Annual Review of Physical Chemistry, 58, 35-55, 2007.  
Tian, T. and Burrage, K., "Binomial leap methods for simulating stochastic chemical kinetics", Journal of Chemical Physics **121**, 10356-10364, 2004.  
Burrage, K., Tian, T. and Burrage, P.M., "A multi-scaled approach for simulating chemical reaction systems", Progress.

## **Random walks and electrical resistance on graphs.**

**Supervisor:** Greg Markowsky

**Co-supervisor:** To be Advised

**Background:** There is a beautiful and well-known connection between the properties of electricity in a circuit and the properties of a walker moving randomly amongst the vertices of a graph. An excellent source concerning this is Doyle and Snell's book, given in the references. In order to be able to make use of this, it is convenient if the graph or circuit in question possesses some degree of symmetry. For infinite graphs it is therefore logical to study objects such as grids, and for finite graphs it is logical to study particularly regular graphs, such as the *distance-regular graphs*.

**Objectives:** The first step will be for the student to thoroughly understand the material presented in the references. From there, we may move in a number of directions, depending on the underlying interest of the student. For example, with a student interested in physics or random processes we may consider some deeper aspects of the problems concerning grids, whereas for a student interested in combinatorics we may concentrate on distance-regular graphs.

### **Expectations:**

- Understanding the material in the references.
- Writing some computer code in order to make and test conjectures.
- Conjecturing and proving some new results, be they publishable or not.

### **Assumed Knowledge:**

Some basic probability and/or combinatorics would be helpful, but is not essential.

### **Reading:**

- Doyle, P. and Snell, J. 1992 *Random Walks and Electric Networks*, MAA.
- Biggs, N. 1993 *Potential theory on distance-regular graphs*. *Combinatorics, Probability and Computing* 2, p. 243-255.
- Biggs, N. 1997 *Algebraic Potential Theory on Graphs*, *Bulletin of the London Mathematical Society*, Volume 29, Number 6, p. 641-682.

## Percolation

**Supervisor(s):** Greg Markowsky

**Co-supervisor:** To be Advised

**Background:** In the last few decades, percolation has become one of the hottest topics in probability theory. To put it simply, suppose that in a given graph we remove each of the edges with a fixed probability  $p$ . We then may ask about the probability of moving from one side of the graph to the other or from one point to another, using only the remaining edges.

**Objectives:** The student must first understand the basics of this subject, and then we will decide together how to proceed. A student with a solid background in complex analysis may be interested in learning about a recently discovered connection between percolation and conformal mappings, called *SLE*. With a student more interested in combinatorics, however, we may study this process on finite graphs.

### Expectations:

- Understanding the prerequisite material.
- Writing computer code in order to understand existing theorems.
- Looking at some problems involving the future direction of this field.

### Assumed Knowledge:

Basic probability theory.

Some complex analysis would also be nice.

### Reading:

- Grimmett, G. 1999 *Percolation*, Springer.

## Stochastic simulation and control of Markov chains

**Supervisor:** Boris Miller

**Co-supervisor:** To be advised

Background: There is a growing interest in modeling and control of stochastic systems described by controllable Markov chains. One of the reasons is that the optimization of such systems is relatively simple and can be made with the aid of optimization tools created for systems described by ordinary differential equations. There are numerous applications of this methodology including, but not exhausting: the transmission control of data in the Internet, dynamic search, restricted resources management etc.

Project outline: This project commences with the implementation of the controlled Markov chains for dynamic search. Then you will implement more efficient methods such as the dynamic programming, constrained optimization based on solution of dual optimization problems, numerical optimization and statistical simulation with the aid of MathLab and Maple. There are two possible projects under this topic.

(1) You may study the dynamic search problem for the moving target with multiple possible locations with the aid of the fast searching system. The results can be applied to the real search problems arising in application of Unmanned Aerial Vehicles. An interesting question is the strategy of search.

(2) You may study effective simulation techniques, including the implementation of stochastic simulation on high performance computers, to simulate the estimation and optimization procedures for systems described by controlled Markov chains.

### References

Elliott, R.J., Aggoun, L and Moore, J.B. “Hidden Markov Models. Estimation and Control” Springer Verlag, New York, 1995.

Miller, B. “Optimization of queuing systems via stochastic control.” *Automatica*, 45, 2009, pp. 1423–1430.

Miller, B., K. Avrachenkov, Stepanyan, K. and Miller, G. “The problem of the optimal stochastic control of a data flow with incomplete information. *Problems of Information Transmission.*” v. 41 n 2, 2005, pp. 150–170.

## The Travelling Salesman Problem and understanding what makes it hard

**Supervisor:** Kate Smith-Miles

**Co-supervisor:** Davaa Baatar

Background: The Travelling Salesman problem (TSP) aims to find the shortest path that connects a set of cities and returns to the starting city. It is an example of a combinatorial optimisation problem, with many practical applications such as logistics and transportation planning, telecommunications routing, and is known to be NP-hard. What we don't know is how the location of the cities and their statistical properties affects the relative hardness of a particular instance of the TSP for different algorithms or methods.

Project outline: This project commence with a study of the statistical properties of TSP instances that are known to affect the hardness of the problem. Much work has been done of identifying parameters that control the easy-hard transition. You will then generate hundreds of TSP instances, solve them using a variety of methods, and then examine the correlation between the characteristics of the instances and the performance of algorithms. Our goal is to be able to explain why some algorithms perform well for some TSP layouts (distribution of cities) and poorly for others. **Please note:** Many other projects are available with Kate Smith-Miles which take a similar approach to different optimisation problems, such as graph colouring, knapsack, bin-packing, timetabling, etc.

### References:

Smith-Miles, K.: “Cross-disciplinary perspectives on meta-learning for algorithm selection” *ACM Computing Surveys* 41(1) (2008)

Gent, I., Walsh, T.: “The TSP phase transition” *Artificial Intelligence* 88(1-2), 349\_358 (1996) **continued**...

Johnson, D.: “Experimental analysis of heuristics for the ATSP” Chapter 10 in: *The Travelling Salesman Problem and Its Variations.* G. Gutin, AP Punnen (2002)

## **Modelling Stem Cell Decision Making**

**Supervisor:** Kate Smith-Miles

**Co-supervisor:** Tianhai Tian

Background: A stem cell is a special kind of cell that has potential to differentiate into other kinds of cells. The mechanisms controlling its decision to stay as a stem cell, or to differentiate, is little understood. If we can model this process mathematically, then we can make predictions that can be tested in the lab (the Monash Stem Cell team are a world-leading group, and this project involved collaboration with them). Some mathematical approaches to modelling stem cell decision making include using coupled differential equation to model at the molecular level, and statistical branching processes to model the broader network of regulatory genes.

Project outline: This project will make a contribution to modelling stem cell decision making based on experimental data from the Monash Stem Cell lab. The approach taken may be based on dynamical systems (molecular level modelling), statistical branching processes (gene network level modelling), or ideally a hybrid of both approaches.

### **References:**

Huang, S., Guo, Y.P., May, G., and Enver, T., "Bifurcation dynamics in lineage-commitment in bipotent progenitor cells" *Developmental Biology*, vol. 305(2), pp. 695-713, 2007.

Balleza, E., Alvarez-Buylla, E.R., Chaos, A., Kauffman, S., Shmulevich, I. & Aldana, M., "Critical dynamics in genetic regulatory networks: examples from four kingdoms", *PLoS One*, vol. 3(6), 2008.

## **Mathematical modelling of regulatory networks based on microarray gene expression data**

**Supervisor:** Tianhai Tian

**Co-Supervisor:** To be advised

Background: The availability of various "omics" datasets creates a prospect of performing the study of genome-wide genetic regulatory networks. However, one of the major challenges of using mathematical models to infer genetic regulation from microarray datasets is the lack of information for protein concentrations and activities. To address this issue, we have designed a more sophisticated modelling framework together with the corresponding inference methods to accurately estimate genetic regulation from "omics" datasets [1]. The next step is to develop specific mathematical models based on microarray gene expression datasets by using our proposed methods [1, 2].

Project outline: This project is designed to use nonlinear mathematical models to infer genetic regulation from microarray gene expression data. We have predicted the 317 putative p53 target genes were supported by DNA sequence analysis. You will use our pre-processed microarray dataset that include more than 8000 genes to derive the connections between these 317 target genes and design a mathematical model to describe the dynamic properties of genetic regulation.

### **References:**

Wang J and Tian T, "Quantitative model for inferring dynamic regulation of the tumour suppressor gene" *p53, BMC Bioinformatics*, 11(1), 36, 2010.

Tian T, "Stochastic models for inferring genetic regulation from microarray gene expression data, *BioSystems*", 99(3), 192-200, 2010.

## **Mathematical modelling of cancer therapy**

**Supervisor: Tianhai Tian**

**Co-Supervisor: To be advised**

Background: Mathematical models for cancer therapy have been designed to describe the response of tumour cells to either radiotherapy or chemotherapy [1]. For example, the Linear-Quadratic (LQ) model and the logistic model are the widely used models for cell killing during the therapy and cell repopulation after the therapy, respectively. Parameters in the LQ model have been estimated for a number of cancer types based on the experimentally measured datasets [2, 3]. The key issue in the modelling for cancer therapy is drug resistance. Since the probability of gene mutation leading to drug resistant is small, more sophisticated mathematical models are needed to accurately reflect the dynamics of tumour therapy response.

Project outline: this project will develop mathematical models to describe the cancer cell population dynamics during and after radiotherapy or chemotherapy. In particular you will find the important mechanisms to regulate the probability of gene mutation and their influence on the drug resistance. Based on the experimental dataset generated from the University of Queensland, it is expected that you may find a critical solution to improve the effects of cancer therapy.

### **References:**

- Tian T, Olson S, Whitacre JM, Harding A., "The origins of cancer robustness and evolvability" *Integrative Biology* (Cambridge). 2011 3(1):17-30.
- Garcia, L.M., et al., "Fitting the linear-quadratic model to detailed data sets for different dose ranges". *Phys Med Biol*, 2006. **51**(11):2813-23.
- McAneney, H. and S.F. O'Rourke, "Investigation of various growth mechanisms of solid tumour growth within the linear-quadratic model for radiotherapy". *Phys Med Biol*, 2007. **52**(4):1039-54.

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